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NEW EXACT SOLUTIONS IN STANDARD INFLATIONARY MODELS

S.V.Chervon, V.M.Zhuravlev, V.K.Shchigolev

Department of Theoretical and Mathematical Physics, Ulyanovsk State University,
42, Leo Tolstoy Str., 432700 Ulyanovsk, Russia Tel/FAX:007-8422-344338;
E-mail: chervon@themp.univ.simbirsk.su

Abstract

The exact solutions in the standard inflationary model based on the self-interacting scalar field minimally coupled to gravity are considered. The shape's freedom of the self-interacting potential $V(\phi)$ is postulated to obtain a new set of the exact solutions in the framework of Friedmann-Robertson-Walker Universes. The general solution was found in the case of power law inflation. We obtained new solutions and compared them with obtained ones earlier for the exponential type inflation.

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1 Introduction

Cosmological inflationary models can be understood as the self-consistent system of Einstein's and nonlinear (scalar, chiral, gauge) fields' equations. Inflationary scenarios are usually based on this type system of equations and contain an analysis of physical reasons which are the sources of an extremely quick expansion (inflation) of the Universe. As a rule the inflationary scenarios are connected with the phase transitions in the framework of quantum theory of scalar field of a finite temperature as well as with spontaneous violation and restoration of the gauge symmetry and another physical phenomenon.

Various type of approximations and numerical methods have been actively used in analysis of inflationary models and brought in the early 80-th the bright success in the explanation of the long standing problems of the Big Bang model such as a horizon, monopolies, large scale structure formation [1]- [5]. Nevertheless now we find out the necessity to make sure in used approximations to match the inflationary model to large scale structure of the Universe. The best way to compare numerical or approximated results with real ones based on the exploitation of the exact solutions.

In the work by A.Guth [1] the inflationary solution has been obtained with the potential $V(\phi) = const$. The exponential expansion of the Universe as an example of the exact solution has been found by G.Ivanov [6] for a nonlinear scalar field with the potential $V(\phi) = \frac{\mu}{2}\phi^2 - \frac{\lambda}{4}\phi^4$ in the framework of spatially-flat Friedmann-Robertson-Walker (FRW) spaces and has been interpreted as the Universe started from a quasivacuum state of matter. Exact solutions of the power law type inflation have been obtained for Liouville non-linearity $V(\phi) = me^{-\lambda\phi}$ [7] (see also [8] and references quoted therein). The Liouville-type and some other non-linearities have been investigated and some exact and asymptotic solutions are presented in [9]. Classical de Sitter type solutions was obtained in [10]. New classes of exact solutions have been found in [11] by taking the scalar field as the function of time $\phi = \phi(t)$ and then determining the evolution of the expansion scale factor $K(t)$ and the potential $V(\phi(t))$ from it. This approach was applied in works [12],[13] as well.

In [14],[15] the exact solutions have been obtained by taking, first, the scalar factor as the function of time $K = K(t)$ and then determining the evolution of the potential $V = V(t)$ and the evolution of a scalar field $\phi = \phi(t)$, the dependence between V and ϕ being, in general, parametric one. This approach called as the method of 'fine turning of the potential' [15].

Another point have been presented in [16], where exact general solutions were found for a single scalar field interacting through an exponential potential in the framework of background field equations for the Arnowitt-Deser-Misner (ADM) formalism. Approximate analytic solutions for slowly evolving multiple scalar fields are obtained also. The Hamilton-Jacobi theory for long-wavelength inhomogeneous universes are investigated in [17] in the framework of ADM formalism. Exact inhomogeneous solutions for Yang-Mills field minimally coupled to gravity have been obtained recently in [18].

In this letter we use the method of ‘fine turning of the potential’ [15] for obtaining new exact solutions in the case of the standard inflationary model. New set of exact solutions for exponential and power law expansion (inflation) are obtained. The generalised Barrow’s solutions [11] are noted.

It should be mentioned here that the self-interacting scalar field theory is just the simplest one among the basic models of quantum field theory [19]. More general approach, as was mentioned above, is based on the GUT coupled to gravity [20],[18] and can be reduced in special cases to the chiral inflationary model [21], [22].

2 Basic equations and the method

The inflationary scenario [1] as well as its first modifications [4],[5] are based on the effective theory of self-interacting scalar field ϕ , minimally coupled to gravity, with the action

$$S = \int \sqrt{-g} d^4x \left\{ \frac{R + 2\Lambda}{2\kappa} + \frac{1}{2} \phi_{,i} \phi_{,k} g^{ik} - V(\phi) \right\}. \quad (1)$$

The cosmological constant includes in the action (1) because of its importance during the inflationary stage.

Let us consider the standard inflationary model (1) in the framework of the Friedmann-Robertson-Walker’s metric

$$dS^2 = dt^2 - K^2(t) \left(\frac{dr^2}{1 - \epsilon r^2} + r^2 (d\theta^2 + \sin^2 \theta d\varphi^2) \right). \quad (2)$$

Here $\epsilon = -1, 0, +1$ corresponds to open, spatially-flat and closed Universe respectively.

An important role in inflationary scenarios belongs to the effective potential of self-interaction $V(\phi)$. The form of the $V(\phi)$ reflects the physical phenomenon at the very early Universe: cosmological phase transitions and the symmetry restoration at high temperatures T in GUTs. It is well known that the form of a potential is changed while phase transitions occur and a temperature increases [2],[3]. The potential depends on the temperature and this dependence is due to quantum one-loop corrections in finite temperature field theory.

Thus the form of the effective scalar potential depends on the type of a field theory, putting into the physical basis, and tends to change when physical phenomenon occur in the development of a cosmological time t .

One more restriction on the form of the potential $V(\phi)$ came from the fine turning procedure [23]. As an example one can consider the situation with the Coleman-Weinberg potential [24] discussed in [27].

In spite of the restrictions above we can come to the conclusion, that the form of the effective potential $V(\phi)$ does not fix and is subjected to change with the evolution of the

Universe. Putting into basis possible variations of the $V(\phi(t))$ we can ask a question:”What kind of the $V(\phi(t))$ admits the exact solutions with an exponential or power law expansion of the FRW Universe?” The answer to this question will give us an explicit form of the potential which leads to the given rate of the expansion of the Universe.

The system of Einstein’s and nonlinear scalar field equations, corresponding to the model (1) in the FRW spaces (2), reads

$$\frac{K_{44}}{K} + \frac{2K_4^2}{K^2} + \frac{2\epsilon}{K^2} = -\Lambda + \kappa V(\phi), \quad (3)$$

$$-\frac{3K_{44}}{K} = \Lambda + \kappa(\phi_4^2 - V(\phi)), \quad (4)$$

$$\phi_{44} + 3\frac{K_4}{K}\phi_4 + \frac{dV(\phi)}{d\phi} = 0. \quad (5)$$

References on a limited number of the exact solutions for the system (3)-(5) is given above.

Considering the equations (3)-(5) one can find that the last equation (5) is the differential consequence of the (3) and (4). Therefore the following analysis will use just Einstein’s equations (3) and (4), which can be reduced to the form where the functions $V(t) \equiv V(\phi(t))$ and $\phi(t)$ are expressed through the function $K(t)$ and their derivatives:

$$V(t) = \frac{1}{\kappa} \left(\Lambda + \frac{K_{44}}{K} + \frac{2K_4^2}{K^2} + \frac{2\epsilon}{K^2} \right), \quad (6)$$

$$\phi(t) = \pm \sqrt{\frac{2}{\kappa}} \int \left(\sqrt{-\frac{d^2 \ln K}{dt^2} + \frac{\epsilon}{K^2}} \right) dt + \phi_0, \quad (7)$$

In (7) ϕ_0 is a constant of integration.

By giving the rate of the expansion as the function for a scale factor on time $K = K(t)$, we can find the functions $\phi(t)$ and $V(t)$ which are necessary for chosen type of the Universe’s evolution. It is obvious, that the pair of the function (6) and (7) gives the parametric dependence $V = V(\phi)$. In some cases, after calculation of the right hand sides in (6), (7), it is possible to find the explicit dependence $V = V(\phi)$ by eliminating t .

3 Exponential type solutions

Let us consider the case when the scale factor $K(t)$ of the Universe grows up very fast by the exponential type law.

We have to mention about the simplest solutions of the exponential type in the form $K(t) = K_0 e^{\lambda t}$, where K_0 and λ are constants.

- For the spatially-flat Universe ($\epsilon = 0$) one can find $\phi = \phi_0 = \text{const}$; $V(\phi) = V_0 = \text{const}$; $\Lambda = 0$ or $\Lambda \neq 0$. This solution corresponds to slow-roll regime [2].
- Closed Universe ($\epsilon = 1$) also admits an exponential expansion if the potential takes the form
$$V(\phi) = \frac{1}{\kappa}(\Lambda + 3\lambda^2 + \frac{2\epsilon\lambda^2}{K_0^2 q^2}(\phi - \phi_0)^2)$$

In the chiral inflationary model [21], [22] it was found the solutions where $K(t) \propto \cosh^\alpha t$ ($\sinh^\alpha t$) and $K(t) \propto \cos^\alpha t$ ($\sin^\alpha t$), with $\alpha = \frac{1}{3}$ or $\alpha = 1$. We will investigate the possibilities to obtain the same rate of expansion in standard inflationary model (1). Therefore we choose the scalar factor of the Universe in the form

$$K(t) = K_0 \cdot \text{ch}^\alpha\{\lambda t\} \quad (8)$$

or

$$K(t) = K_0 \cdot \text{sh}^\alpha\{\lambda t\} \quad (9)$$

where λ -constant. The scalar factor (9) have been obtained for the spatially flat FRW Universe in [11]. For the sake of simplicity let $\alpha = 1$.

In the case (8) the scale factor $K(t)$ is finite in the initial moment $t = 0$. In the case (9) the scalar factor equals to zero and we have the initial singularity. In the both cases the solutions for $\phi(t)$, $V(t)$ and $V(\phi)$ can be obtained explicitly. Starting from the case (8) one can find

$$\begin{aligned} \phi(t) &= \mp \frac{q}{\lambda} \text{arctg}\left\{\frac{1}{\text{sh}(\lambda t)}\right\}, \\ V(\phi) &= \frac{1}{\kappa} \left[\Lambda + 3\lambda^2 - q^2 \kappa^2 \cos^2 \frac{2\lambda\phi}{q} \right], \end{aligned} \quad (10)$$

where $q = \frac{1}{\kappa} \sqrt{\frac{2\epsilon}{K_0^2} - 2\lambda^2}$. In the case of (10) the dependence V on the field ϕ is the periodic one (see fig. 1 and 2). The evolution of the Universe begins from finite radius which then exponentially expands to infinite one. This situation corresponds to a rolling from unstable equilibrium points of the maxima to the stable equilibrium points of the potential's minimum on the plane $V - \phi$.

In the case (9) the solution can be presented as

$$\begin{aligned} \phi(t) &= \pm \frac{q}{2\lambda} \ln \frac{\text{h}(\lambda t) - 1}{\text{ch}(\lambda t) + 1}, \\ V(\phi) &= \frac{1}{\kappa} \left[\Lambda + 3\lambda^2 + q^2 \kappa^2 \text{sh}^2 \frac{2\lambda\phi}{q} \right], \end{aligned} \quad (11)$$

where $q = \frac{1}{\kappa} \sqrt{\frac{2\epsilon}{K_0^2} + 2\lambda^2}$. The Universe evaluates from the singular state, corresponding to infinity large value of the field ϕ , to the state with infinitely large radius, which corresponds to the single local minimum of the potential $V(\phi)$. These solutions coincide with Barrow's one [11] when $\epsilon = 0$.

Let us mention also that the potential (11) leads to one more exact solution, corresponding to harmonic variation of the scale factor

$$K(t) = a_1 \sin\{\lambda_1 t\}$$

where a_1 and λ_1 are special values of parameters. The solution is described by

$$\begin{aligned} \phi(t) &= \pm \frac{q}{2\lambda_1} \ln \frac{1 - \cos(\lambda_1 t)}{1 + \cos(\lambda_1 t)}, \\ V(\phi) &= \frac{1}{\kappa} \left[\Lambda - 3\lambda_1^2 + q_1^2 \kappa^2 \operatorname{ch}^2 \frac{2\lambda_1 \phi}{q_1} \right], \end{aligned} \quad (12)$$

where $q_1 = \frac{1}{\kappa} \sqrt{\frac{2\epsilon}{a_1^2} + 2\lambda_1^2}$. This type of solutions called in [11] as a trigonometric counterpart to the solution (9).

The potentials (11) and (12) coincide with the additive constant accuracy if $\lambda_1 = \lambda$, $q_1 = q$. The last regime (12) corresponds to periodical crossing of the singular state.

Let us mention now that all solutions presented in this sections exist independent off the type of the Universe: open, spatially-flat or closed. The type of the Universe depends on the sign of subradical's expression $\frac{2\epsilon}{a_1^2} \pm 2\lambda_1^2$. If $\frac{2\epsilon}{a_1^2} \pm 2\lambda^2 > 0$, then the above cases are realized. If $\frac{2\epsilon}{a_1^2} \pm 2\lambda^2 < 0$, then the cases (10) and (11) are changed by places. The exceptional case $q = 0$ corresponds to zero or any constant value of the field.

Let us note again that the solutions (11) and (12) can be considered as the generalisation of Barrow's solutions [11] for the case of open and closed universes.

4 Power law inflation

For the power law inflation the scale factor can be presented in the form

$$K(t) = K_0 t^m.$$

The integral in the right hand side of (7) can be calculated explicitly. Therefore we can find ($m \neq 1$)

$$V(\phi(t)) = \frac{m}{\kappa} \left(\frac{\Lambda}{m} - t^{-2}(3m - 1 - 2\alpha t^{-2m+2}) \right) \quad (13)$$

$$\phi(t) = \pm \sqrt{\frac{1}{2\kappa}} \frac{m}{1-m} \left\{ 2\sqrt{1 + \alpha t^{-2m+2}} + \ln \left(\frac{\sqrt{1 + \alpha t^{-2m+2}} + 1}{\sqrt{1 + \alpha t^{-2m+2}} - 1} \right) \right\} + \phi_0, \quad (14)$$

where $\alpha = \epsilon K_0^{-2}/m$.

- As was mentioned in [27] for the spatially-flat Universe ($\epsilon = 0$) the solution for arbitrary m has the form

$$\phi = \pm\sqrt{2m/\kappa} \ln t + \phi_0, \quad V(t) = \kappa^{-1}(\Lambda + (m + 3m^2)t^{-2}). \quad (15)$$

Eliminating t we find an exponential dependence V on ϕ

$$V(\phi) = \kappa^{-1}\{\Lambda + (m + 3m^2) \exp(-\sqrt{2\kappa m^{-1}}\{\phi - \phi_0\})\}, \quad (16)$$

what is usually the definition of the power law inflation.

- In the case of open and closed Universe ($\epsilon \neq 0$) it is possible to find an explicit dependence V on ϕ just for some values of m . For example, if $m = 1$ (in this case the formulas (13) and (14) do not work)

$$V(\phi) = \Lambda/\kappa + \kappa^{-1} \exp\left\{-\frac{2(\phi - \phi_0)}{\pm\sqrt{2/\kappa}\sqrt{1 + \epsilon K_0^{-2}}}\right\}, \quad (17)$$

- General behaviour all functions $K(t)$, $V(\phi)$, $\phi(t)$ are common for any $m > 1$. The figure 3 illustrates the behaviour of the functions in the case $m = 5$.

5 Conclusions

The idea that the shape of the potential $V(\phi)$ in cosmological inflationary models does not fixed allows us to find new exact solutions for exponential type inflation. In the case of power law inflation it was found the general solution. Both cases are considered for FRW Universes. The following task will be to find an appropriate theory for obtaining the relevant shape of the effective potential $V(\phi)$.

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Figure captions:

We use a relative units for time t , the scalar field ϕ , the potential $V(\phi)$ and scalar factor K for the sake to illustrate the general behaviour of all functions.

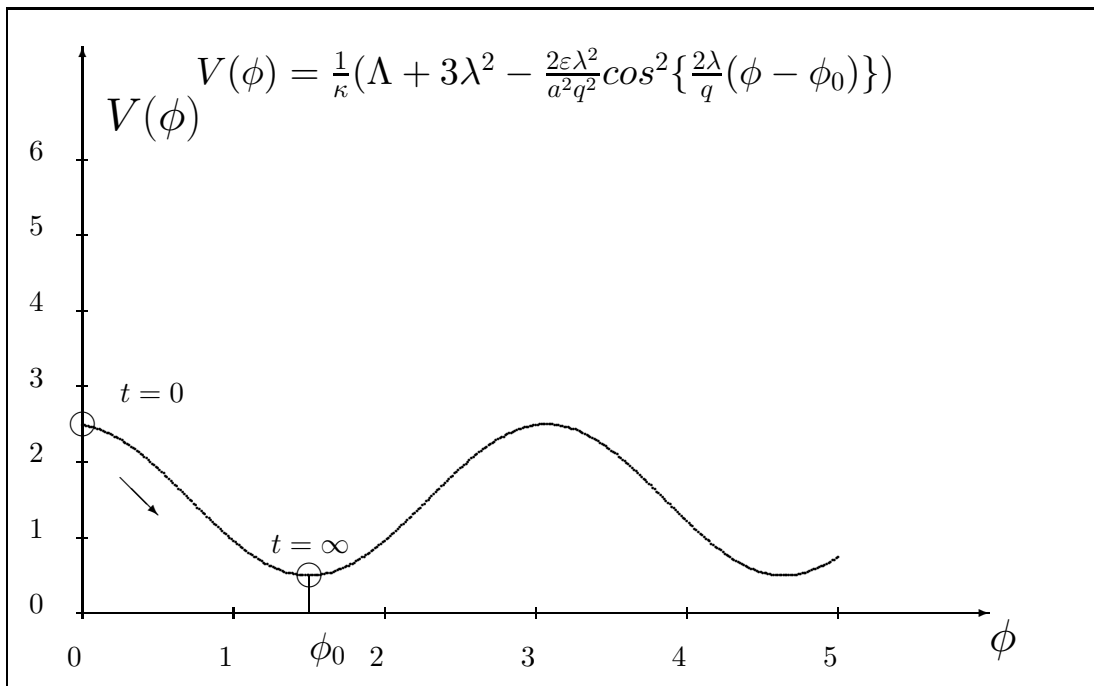


Figure 1: The shape of the periodic potential.

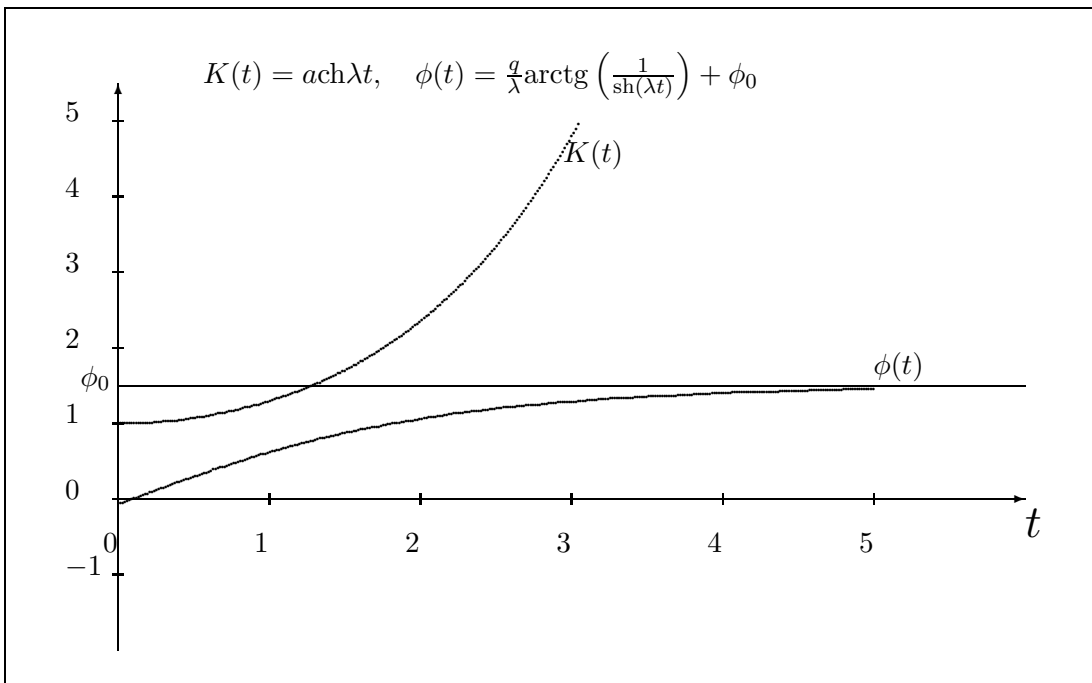


Figure 2: The scalar factor $K(t)$ and the scalar field $\phi(t)$ for the periodic potential.

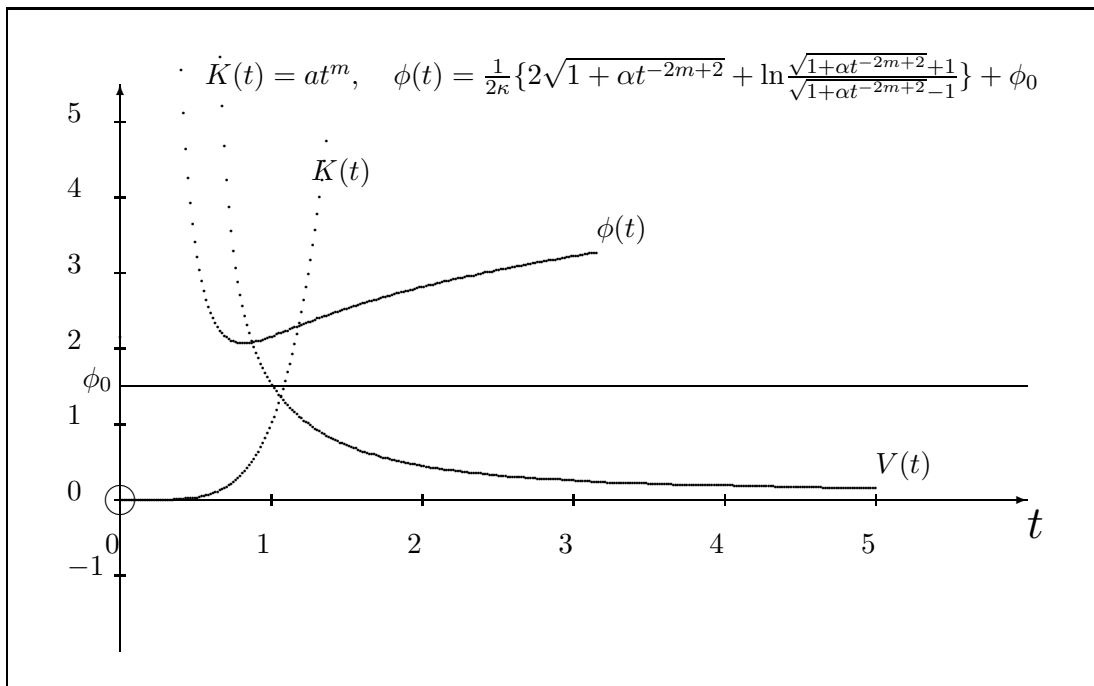


Figure 3: Power law inflation with $m=5$