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Matter and Space. New Theory of Fields and Particles

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Abstract—The paper presents a theory giving a unified geometric description of space and matter on the basis of a new concept related to general relativity (GR). The theory is built on the basis of a critical analysis of GR. The principle of materiality of space is introduced. The description of matter is based on the idea of space as a three-dimensional material hypersurface embedded in a four-dimensional Euclidean space. Matter particles are associated with extended areas of the material hypersurface, and their properties, such as charge and mass, with topological and geometric properties of this hypersurface. The central place in the mathematical apparatus for describing the material hypersurface itself and matter particles is played by marker fields, which are similar in essence to hydrodynamic markers used in classical hydrodynamics. Based on the theory of marker fields, questions of the topological structure of particles and connection between the electric charge and the topology of a material hypersurface are discussed. The mass of particles is represented as a property of the material hypersurface itself and has the meaning of gravitational and inertial mass at the same time. The fields, gravitational and electromagnetic, are properties of the material hypersurface geometry expressed in terms of marker fields. To describe the dynamics of particles, the geometric principle of averaging is introduced, which, as a result, leads to the equations of Newtonian mechanics and quantum theory.

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1. INTRODUCTION

The problems to be discussed in this paper are related to the fundamental concepts of modern physics: mass, electric charge and their connection with the structure of physical space as a material object. The approach to the description of space as a material object in the form of a 3D smooth hypersurface embedded in enclosing 4D Euclidean space is presented in the papers [1–9].

In [8, 9], the principle of the materiality of space was formulated as an alternative to the general approach of special (SR) and general (GR) relativity. The need to adopt the principle of the materiality of space was justified in these papers by the fact that the space-time of SR and GR, being an immaterial object, is nevertheless endowed with physical properties measurable in the experiment. In SR, these are the properties of changing the length scale and the clock rate at transitions from one inertial reference frame to another. In GR, these are additional properties of space-time curvature that determine the properties of the gravitational field. As a result, contradictions and paradoxes appear in the physical theories, for example, the twin paradox in SR and the gravitational field energy problem in GR.

The earlier papers [1–7] discussed a new approach to the description of electric charge, mass and the related fields, the electromagnetic and gravitational ones, as well as quantum theory. The basic approach to the description of all these physical concepts was the representation of space as a 3D hypersurface $\mathcal{V}^3$ in enclosing 4D Euclidean space $\mathcal{W}^4$. The shape of this hypersurface is determined by the height function $\mathcal{F}(\mathbf{x}, t)$ according to the equation

$$w = \mathcal{F}(\mathbf{x}, t), \tag{1}$$

where $\mathbf{x} = (x^1, x^2, x^3)$ and w are Cartesian coordinates in $\mathcal{W}^4$, connected with the distinguished hypersurface $\mathcal{P}^3 \in \mathcal{W}^4$ and the direction orthogonal to it, respectively. The materiality of the hypersurface $\mathcal{V}^3$ is described in this approach by introduction of markers for points of this hypersurface, which are analogous to markers used in hydrodynamics. As explained in [8, 9], markers are the simplest and most natural way of tracking material objects of almost any type. The present paper proposes an updated presentation of the theory which was earlier called the *Topological theory of fundamental fields* (TTFP).

We will begin the description of the new approach with the statement of the materiality principle for physical objects, which was formulated in [8, 9], with application of this physical principle to physical space.

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Next, the general approach will be considered, the theory of marker fields, which seems to be the most adequate way to describe material objects of any nature. The theory of marker fields for describing material space in the form of a hypersurface in $\mathcal{W}^4$ was also considered in general terms in [8, 9]. It should be noted that the marker theory is one of the standard ways to describe hydrodynamic flows of gases and fluids. What is new in the proposed theory is the use of marker theory to describe gravitational and electromagnetic fields [1–9].

We will begin with using marker fields to describe the flows of gas and dust in their own gravitational field, which is the main object of research in the problems of astrophysics. It will be shown how such an approach is transferred to the problems of describing the very structure of space and the particles of matter that we perceive as elementary particles. Such particles obey the laws of quantum physics, which is also not free from difficulties with a rational interpretation of its conclusions (see, e.g., [10]). Therefore, in the article we will briefly focus on how the marker theory allows us to rationally explain the basic ideas underlying quantum theory, without resorting to postulates on the impossibility of understanding the microcosm with the help of our experience of living in the macrocosm of classical mechanics.

2. THE MATERIALITY PRINCIPLE FOR PHYSICAL OBJECTS

The necessity to introduce, in modern physical theory, the seemingly obvious **the materiality principle for physical objects** studied with the help of instruments in physical experiments, has become urgent due to the difficulties experienced by all basic physical concepts—SR, GR, and quantum theory. This principle is formulated as follows: *Any object that can be detected by physical devices and has properties detected in a physical experiment must be a material object, i.e., it must possess the fundamental properties of matter—mass and energy.*

The existence of difficulties in SR, GR, and quantum theory has been discussed throughout the XX century by many authors, see, for example, the papers by Fock [11], Brulluin [12] on SR and GR, and Sudbury's paper [10] on a strict formulation of the postulates of quantum theory. An analysis of these difficulties [8, 9] indicates that the main source of problems of modern physical theories is violation of the materiality principle for physical objects while describing the structure and dynamics of the main object of SR and GR, the Einstein space-time. Space-time itself in SR and GR is an immaterial object, just like space in classical mechanics.

In classical mechanics, a measurement of distances is possible only between individual material bodies or their parts. The general idea of three-dimensional space in experiment and theory arises as a cumulative mathematical model that adequately combines the entire set of distances between material objects obtained as a result of experiments. The same situation is transferred to space-time in SR and then GR, in which space is necessarily combined with time. Unlike classical mechanics, in which space and time are absolute and devoid of any physical properties, except for the actual length and duration, space-time in SR and GR is endowed with peculiar physical properties. In SR, it is the change in length and time scales at a transition from one reference frame to another, which are detected in the experiment, and with the help of which a number of actually observed physical phenomena are explained.

In GR space-time is additionally endowed with the property of curvature that serves to explain such a phenomenon as gravity. Einstein's idea that the very properties of space-time determine the phenomena that we call the gravitational field seems to be one of the most important ideas in physics of the 20th century. A. Einstein gave a concrete physical meaning to guesses of the 19-th-century scientists Lobachevsky, Gauss and Clifford, that the properties of non-Euclidean space can explain many phenomena in the observable world, up to the geometric essence of matter itself, as proposed in general terms by Clifford [13]. Without this idea, it is difficult to understand what are gravitational and electromagnetic fields are. When we observe the orbital motion of planets around the Sun or the Moon around Earth, we do not see any specific substance that causes the planets to move, deviating from uniform and rectilinear motion.

Similar questions may be addressed to the electromagnetic field as well. The fields have no color, taste, or flavor. If we assume that the fields are properties of space itself, then all problems disappear by themselves. However, at the same time, another problem arises, the problem of materiality of space itself.

This problem consists in the fact that by endowing an immaterial object with physical properties, we refuse to specify a physical mechanism that is responsible for a particular physical phenomenon. This leads to various paradoxes. The essence of such inadequacy of the theory was explained by the example of the twin paradox and the effects associated with atmospheric muons in [8, 9]. From a mathematical point of view, SR and GR look absolutely perfect. Nevertheless, mathematics itself often points out contradictions when trying to link immaterial objects with real physical processes. For example, such a situation occurs in GR when trying to determine the energy of the gravitational field, described by the properties of non-Euclidean pseudo-Riemannian space-time. The inability to attribute any specific energy to the gravitational field in the form of Einstein space-time makes it impossible to construct a quantum theory of gravity.

Mathematically, this problem reduces to the fact that the analogue of the energy-momentum tensor for a gravitational field is not a tensor. It is this fact that leads to the problem with the energy of the gravitational field from a mathematical point of view.

The principle of materiality of space excludes the occurrence of such problems. At the same time, it does not give direct indications of how one should construct a theory of space and time which would describe the whole set of physical phenomena. To overcome this difficulty, we will use, as already noted in the introduction, the theory of markers and marker fields.

3. CLASSICAL FORCE FIELDS AND MARKER FIELDS

A typical problem that occurs in astrophysics when describing various evolving structures and objects, such as stars or galaxies, is the problem of describing gas and dust flows in their own gravitational field. In all problems not related to superdense objects, such as neutron stars, black holes and quasars, GR effects are usually neglected, i.e., the space is considered as a 3D absolute space of classical mechanics with an independent uniform time flow at all points of space. In most of these tasks, SR effects are also neglected since, for most of these objects, the velocity of dust and gas flows is much less than the speed of light. A basis for describing the medium itself in such problems is a model of a continuous medium, i.e., a medium consisting of material points having the shape of geometric points endowed with physical properties—mass and possibly electric charge, if a plasma is concerned. In reality, these material points mean atoms, and in some problems of galactic and cosmological dynamics, individual stars.

To describe a self-gravitating medium consisting of material points, two approaches can be used, the Lagrange and Euler ones. In the Lagrangian approach, the material points of the medium are fixed using various markers, for example, marking them with the coordinates they had at the initial time instant. In this case, the Newtonian equations of motion are written for each particle separately. However, the Eulerian approach is more often used, it consists in writing equations for each material point that appears at a selected point in space at fixed time. The system of Euler equations describing a flow of gas and dust in its own gravitational field, has the form

$$\mathbf{u}_t + (\mathbf{u}, \nabla)\mathbf{u} = -\frac{1}{\rho_m}\nabla p - \nabla\phi, \tag{2}$$

$$\rho_{m,t} + \mathrm{div}\,(\rho_m \mathbf{u}) = 0, \tag{3}$$

$$\Delta\phi = 4\pi G\rho. \tag{4}$$

Here, $\mathbf{u} = (u^1, u^2, u^2)$ is the vector field of medium flow velocity, $\rho_m(\mathbf{x}, t)$ is its mass density, $\phi(\mathbf{x}, t)$ is the gravitational field potential, $p(\mathbf{x}, t)$ is the pressure, G is Newton's gravitational constant. The first equation, (2), is the Euler equation of continuum dynamics. Equation (3) is the mass conservation equation, and (4) is the Poisson equation for the gravitational potential. To close this set of equations, it i necessary to add an equation of state of this medium, and possibly a heat conduction equation.

Meanwhile, in Euler's approach, it is also possible to pass on to the description of medium dynamics using marker fields. Markers are fields whose values do not change along the current lines. Let $e^a(\mathbf{x}, t)$, $a = 1, 2, 3$ be a set of marker fields. Then, by definition, these fields satisfy the marker transfer equations

$$\frac{de^a}{dt} = \frac{\partial e^a}{\partial t} + (\mathbf{u}, \nabla)e^a = 0, \quad a = 1, 2, 3. \tag{5}$$

This approach is widely used in astrophysical problems when dealing with spherically symmetric flows. In this case, a marker is the mass function $\mathcal{M} = 4\pi \int_0^r \rho r^2 dr$, the amount of mass contained inside a ball of radius r (see, e.g., [14]).

The most important point in this approach for further generalizations is that the gravitational field strength turns out to be closely related to the marker fields, which has served as the basis for the development of the TTFP theory describing matter as such in the form of elements of space topology and geometry. Moreover, all main dynamic parameters of the medium, the velocity field and the density, can also be expressed exclusively in terms of the marker fields. In particular, from the marker transfer equations (5), automatically follows a connection between the marker fields and the velocity field of the medium:

$$u^\alpha = -\frac{\partial e^a}{\partial t}\frac{\partial x^\alpha}{\partial e^a}. \tag{6}$$

Using the marker transfer equations (5), one can obtain a general representtaion of the density that automatically satisfies the mass conservation equation (3). Differentiating (5) in the coordinates, we arrive at the relation

$$\frac{\partial}{\partial t}\frac{\partial e^a}{\partial x^\alpha} + \frac{\partial}{\partial x^\alpha}\left(u^\beta \frac{\partial e^a}{\partial x^\beta}\right) = 0.$$

Multiplying this relation by the derivatives $\partial x^\alpha/\partial e^a$ and contracting the result over the indices α and a, we arrive at the equation

$$\frac{\partial |J|}{\partial t} + \frac{\partial}{\partial x^\alpha}\Big(|J| u^\alpha\Big) = 0, \qquad (7)$$

where J is the Jacobian of the transformation $e^a \to x^\alpha$:

$$J = \det\left(\frac{\partial e^a}{\partial x^\alpha}\right). \qquad (8)$$

Comparing (3) and (7), we conclude that, from a formal viewpoint, the density ρ can be identified with $|J|$. In a more general interpretation, a relation between ρ and $|J|$ may have the form

$$\rho_m = m_0 \mathcal{M}(\mathbf{e})|J|, \qquad (9)$$

where $\mathcal{M}(\mathbf{e})$ is an arbitrary sufficiently smooth function of the marker fields: $\mathbf{e} = (e^1, e^2, e^3)$, and m_0 is a constant of mass dimension that provides the necessary dimensionality of mass density ρ_m. For any such function $\mathcal{M}(\mathbf{e})$, the function ρ_m is a conserved density, which can be verified by a direct check of the relation

$$\frac{\partial \rho_m}{\partial t} + \mathrm{div}\,(\rho_m \mathbf{u}) = 0.$$

The factor $\mathcal{M}(\mathbf{e})$, by its meaning, determines some distinctions in the properties of medium particles, and it will play an important role iin our further interpretations of the gravitational fields.

The equality (9) determines a relationship between the properties of marker fields and the mass density of the medium. Let us now consider how a connection is established between the marker fields and the free fall acceleration due to the force of gravity. To that end, consider the formal identity

$$\frac{\partial e^a}{\partial e^a} = 3, \qquad (10)$$

which holds on the Cartesian map of the marker space $\mathcal{E}^3$. Let us now turn in this identity to spatial coordinates, whose functions are the marker fields. As a result of the point-by-point transformation of the marker space into the coordinates of physical space, $e^a \to x^\alpha$, the identity (10) turns into a relation of the following form:

$$\frac{\partial}{\partial x^\alpha}\left(|J| \frac{\partial x^\alpha}{\partial e^a} e^a\right) = 3|J|. \qquad (11)$$

Since $|J|$ is related to the medium density, it is logical to suggest an interpretation of (11) as the Poisson equation for free-fall acceleration, which has in this case the form

$$g^\alpha = g_0^\alpha + \mathrm{curl}\,\mathbf{z}, \qquad (12)$$

where

$$g_0^\alpha = \frac{4\pi G}{3} m_0 |J| \frac{\partial x^\alpha}{\partial e^a} e^a. \qquad (13)$$

The term curl $\mathbf{z}$ makes it possible to assure that the free-fall acceleration has the standard form of a gradient field. For this purpose, the vector field ζ should satisfy the equation

$$\mathrm{curl\,curl}\,\mathbf{z} = -\mathrm{curl}\,\mathbf{g}_0,$$

where $\mathbf{g}_0$ has the components (13). Thus, all basic elements describing the dynamics of a self-gravitating medium (except for its temperature) can be expressed in terms of the properties of the marker fields e^a.

Note that partially similar constructions can be made for a plasma flow in the magnetohydrodynamic approximation. Indeed, since the charge density ρ_e is a conserved quantity, the following equation must hold:

$$\rho_{e,t} + \mathrm{div}\,(\rho_e \mathbf{u}) = 0. \qquad (14)$$

It follows that the charge density is also related to the marker fields, like the mass density,

$$\rho_e = e_0 \mathcal{Q}(\mathbf{e})|J|. \qquad (15)$$

Here $\mathcal{Q}(\mathbf{e})$ is a quantity characterizing the point charge with the markers $\mathbf{e}$, and e_0 is a dimensional multiplier that ensures the correct dimension of ρ_e. This relation reflects only the fact that the medium consists of material points, additionally endowed with an electric charge. Unlike a real electric charge, the charge of material points is infinitely small and not discrete. Unlike an electroneutral medium, a plasma is subject to both electric and magnetic fields. For the electric field strength, the equation has a form similar to the Poisson equation for the gravitational field strength:

$$\mathrm{div}\,\mathbf{E} = 4\pi \rho_e. \qquad (16)$$

It follows that the electric field strength can be associated with the properties of markers by analogy with (12) and (13). Difficulties in this case arise only with the Lorentz force and the magnetic field. This issue will be considered further on in the framework of the new approach.

The main conclusion that can be drawn from the previous analysis is that for the classical force fields, the gravitational and electric ones, acting on a continuous medium, there is an effective representation through the properties of marker fields $e^a = e^a(\mathbf{x}, t)$, "numbering" the points of the medium. A more general principle, which will be used later, is that *any material object can be represented at a certain level of description as a set of material points whose dynamics is formulated in terms of the dynamics of marker fields*, by definition satisfying

Eqs. (5). This principle will be applied to space itself. This will allow us to construct a closed scheme for explaining the observed properties of fields and particles using geometric and topological properties of 3D space as a hypersurface $\mathcal{V}^3$ embedded in the Euclidean 4D space $\mathcal{W}^4$. This approach represents a new realization of the ideas of Clifford and Einstein about the geometric origin of physical fields and material particles, as well as their properties.

4. MARKER FIELDS AND THE GEOMETRY OF SPACE

As already noted, the first starting point of the new theory, which determines a general scheme for explaining the properties of fields and matter from a geometric point of view, is that 3D physical space has the form of a non-Euclidean 3D hypersurface $\mathcal{V}^3$ embedded in Euclidean 4D space $\mathcal{W}^4$. The space $\mathcal{W}^4$ itself is in the theory an absolute space in which time flows at the same rate at all its points. The difference between this space and the absolute space of classical mechanics is only its dimension.

This space is immaterial. This does not mean that in future theories it will not have to be endowed with any physical properties. But after this space will be supplied with some measurable physical parameters, it will have to acquire the status of a material object.

Since the hypersurface $\mathcal{V}^3$ is recognized as a material object that is allocated in $\mathcal{W}^4$ using the equation (1), its dynamics can be described using marker fields $e^a(\mathbf{x}, t)$, which now "number" not the points of the medium, "located" in space, but the hypersurface points of $\mathcal{V}^3$ themselves. The marker transfer equations in this case will coincide with the Eqs. (17) with the difference that the coordinates $\mathbf{x} = (x^1, x^2, x^3)$ of the markers' positions do not refer to $\mathcal{V}^3$, but to the 3D distinguished hyperplane $\mathcal{P}^3$, for which Eq. (1) is written. The hyperplane $\mathcal{P}^3$ in this case should be considered as a mathematical implementation of an inertial reference frame. Therefore, when working with a non-Euclidean hypersurface $\mathcal{V}^3$, we will use Cartesian spatial coordinates usual for classical mechanics, which is significant for deriving a number of important mathematical relations of the theory.

The use of marker fields in the construction of the theory allows us to immediately introduce several physical parameters that may be related to the properties of matter. To do that, consider again the marker transfer equations (5):

$$\frac{\partial e^a}{\partial t} + (\mathbf{V}, \nabla) e^a = 0, \tag{17}$$

but, for convenience, we will replace the notation $\mathbf{u}$ for the tranfer field with $\mathbf{V}$ with the components V^α, bearing in mind that the transfer field $\mathbf{V}$, specified on the plane $\mathcal{P}^3$, describes a transition of points of the hypersurface $\mathcal{V}^3$ rather than points of a medium.

The first parameter that can be obtained by analogy with the previous constructions is the function $|J|$, the absolute value of the Jacobian of the mapping $e^a \to x^\alpha$, which is calculated by the rule (8). In essence, the value of $|J|$ determines the density of the number of markers or points of the hypersurface $\mathcal{V}^3$ contained in a small neighborhood of each point with coordinates $\mathbf{x}$ on the hyperplane $\mathcal{P}^3$. As follows from (7), this quantity is a conserved density, which allows it to be associated with the mass density of matter if material objects are somehow isolated from the structure of the hypersurface $\mathcal{V}^3$ itself. Following this idea, we can assume that the real mass density corresponding to each point of space $\mathbf{x}$ can be calculated according to Eq. (9), but in which the constant m_0 of mass dimension must be a fundamental constant. In this case, the function $\mathcal{M}(\mathbf{e})$ will reflect the "massiveness" property of each individual point of $\mathcal{V}^3$. This property of massiveness can have various physical realizations, for example, reflect the presence of an nonuniform "thickness" of a material object described in the theory as a material hypersurface $\mathcal{V}^3$.

The second natural property of space as a material object which we obtain using marker fields, is the field of gravity. By analogy with the medium density, we will assume that the function

$$\rho_m = m_0 \mathcal{M}(\mathbf{e})|J|, \tag{18}$$

is the mass density of points belonging to $\mathcal{V}^3$, projected onto $\mathcal{P}^3$. Now $|J|$ is the density of markers, while $\mathcal{M}(\mathbf{e})$ is the "massiveness" property of the points of $\mathcal{V}^3$ itself. For any function $\mathcal{M}(\mathbf{e})$ and the transfer field $\mathbf{V}$, the function ρ_m is a conserved density, as was already pointed out:

$$\frac{\partial \rho_m}{\partial t} + \operatorname{div}(\rho_m \mathbf{V}) = 0. \tag{19}$$

Moreover, choosing as $\mathcal{M}(\mathbf{e})$ different functions, one can enumerate all possible conserved densities corresponding to given $|J|$ and $\mathbf{V}$.

Now, using the identities (10) and (11), we arrive at the Poisson equation

$$\operatorname{div} \mathbf{g} = 4\pi G \mathcal{R}(\mathbf{x}, t) \rho_m \tag{20}$$

with the components

$$\mathrm{g}^\alpha = \mathrm{g}_0^\alpha + [\nabla \times \mathbf{z}]^\alpha, \quad \alpha = 1, 2, 3, \tag{21}$$

where

$$\mathrm{g}_0^\alpha = \frac{4\pi m_0 G}{3} \mathcal{M}(\mathbf{e})|J| e^a \frac{\partial x^\alpha}{\partial e^a}.$$

At that, the function $\mathcal{R}$ has the form

$$\mathcal{R} = 1 + \frac{1}{3} e^a \frac{\partial \ln \mathcal{M}}{\partial e^a}. \qquad (22)$$

The last term in (21), which is essentially a gauge for the gravitational field strength, now plays a different role from that of the similar term in (12). Now there is no reason to believe that the field $\mathbf{g}$ should be a gradient of some potential. However, the term curl $\mathbf{z}$ can provide special conditions for $\mathbf{g}$ at large distances from material bodies, where, according to Newton's theory of gravity, the acceleration of gravity should tend to zero.

Let us make a few remarks on the relations obtained.

It is not difficult to see that if $\mathcal{M}(\mathbf{e})$ is not a constant, then in the right-hand side of the generalized Poisson equation (20), the factor $\mathcal{R}$ emerges before the medium mass, which can be associated with the phenomenon that is currently called the hidden mass or the mass of dark matter. This initially makes the theory suitable for explaining dark matter as a property of space itself. This idea was discussed in more detail in [4, 7].

The second remark is that, in the theory being developed, there is no reason to ensure that this field $\mathbf{g}$(21) is a gradient of some potential. In the problems of classical theory, due to Newton's law of gravity, the gravitational field strength should be a gradient of a potential. But when constructing a theory that connects the gravitational field with the geometric properties of space, there is initially no need to introduce such a requirement. At the same time, a vortex component of this field must be included in the theory as an additional type of action of the real gravitational field, which, under certain conditions, disappears or becomes so small that it turns out to be invisible in experiments. It should be noted that the possible existence of a vortex component of the gravitational field was pointed out by Brillouin in [12]. This assumption arises from a simple comparison of the classical Poisson equation for the gravitational field and the first Maxwell equation for the electric field strength.

Of interest is also the physical meaning of the field $\mathbf{g}_0$, related to the way the Poisson equation emerges in this theory. In contrast to classical mechanics, where the Poisson equation is a direct consequence of Newton's law of gravitation, in the theory being developed, the equation appears as a consequence of the identity (10) for the coordinates e^a in the marker space. This opens up the possibility of giving a visual interpretation of the physical meaning of the gravitational field through the properties of $\mathcal{V}^3$ as a material object. Formally, the mathematical meaning of (20) stems from the identity (10). The initially trivial identity (10), as a result of passing over to coordinates on the hyperplane $\mathcal{P}^3$, designates the topological continuity of numbering of points of the physical hypersurface $\mathcal{V}^3$, and consequently, $\mathcal{V}^3$ itself. But a clearer interpretation can be given after the theory acquires an idea of matter particles.

5. THE ELECTRIC FIELD AND MARKER FIELDS

For an introduction of matter particles to the theory, it is necessary to list their fundamental properties, to be explained and described in terms of the geometry and topology of $\mathcal{V}^3$, linking them with marker fields. First of all, it is necessary to introduce into the theory the idea of an electric charge and the electromagnetic fields associated with it. However, now we cannot simply endow the points of $\mathcal{V}^3$ with the electric charge property, as could be done for a plasma. An important circumstance to be taken into account in the new geometric realization of an electric charge is its discreteness. If we attribute the property of having an integer charge to points of $\mathcal{V}^3$, which is thought of as a smooth hypersurface in $\mathcal{W}^4$, then the density of such a charge will be a discontinuous function everywhere, and the total charge of individual regions of space will change unpredictably. Therefore, the very integer nature of the electric charge indicates that such a quantity should not characterize points of $\mathcal{V}^3$, but rather its individual regions. At the same time, it is necessary to take into account that the electric charge in experiments has spatial localization. The charge is "concentrated" inside individual elementary particles—electrons, protons, etc. Meanwhile, the electron behaves in scattering experiments as a point object, and point scatterers (partons) are found in the structure of nucleons. Taking into account the discreteness of the charge magnitude and the data on charge localization inside the particles, it can be concluded that the charge should be treated as an element of topology of the hypersurface $\mathcal{V}^3$.

In mathematics, integer quantities characterizing the structural features of spatial regions are associated with their topological invariants [20, 24–26]. Therefore, an approach based on the theory of topological invariants of 3D regions of space should be considered as a natural approach for introducing an integer electric charge into the theory, and as it will turn out later, other charge numbers, for example, a baryon charge. Such an idea was proposed in [17, 18].

A way to construct the necessary description of matter particles endowed with discrete integer charges can be seen in the use of differential identities for marker fields, as was done for the gravitational field. The idea of using marker fields to introduce discrete electric charge theory was proposed in [1—5]. The same approach gives a natural description of matter particles as regions of space distinguished in a special way.

By analogy with the identity (10), consider another identity written in the coordinates of the marker space $\mathcal{E}^3$. This identity has the form

$$\frac{\partial}{\partial e^a}\frac{e^a}{|\mathbf{e}|^3} = 4\pi\delta(\mathbf{e}), \tag{23}$$

where $|\mathbf{e}|^2 = s\sum_{a=1}^{3}(e^a)^2$, with the Dirac delta function $\delta(\mathbf{e}) = \delta(e^1)\delta^2(e^2)\delta(e^3)$ having as its carrier the origin of the marker space. In physics, the identity (23) is a formal consequence of the Coulomb law and a definition of the field strength of a point electric charge equal to uniity.

Transforming this identity, as in the case (10), to the coordinates on $\mathcal{P}^3$, we arrive at the relation

$$\frac{\partial}{\partial x^\alpha}\left(\frac{|J|}{|\mathbf{e}|^3}e^a\frac{\partial x^\alpha}{\partial e^a}\right) = 4\pi|J|\delta(\mathbf{e}(\mathbf{x},t)). \tag{24}$$

Here, in the r.h.s., there is an expression containing a δ function of the fields $e^a(\mathbf{x},t)$, which are in turn functions of the coordinates. From the properties of the δ function [15] it follows

$$\delta(\mathbf{e}(\mathbf{x},t)) = \sum_{k=1}^{N}\frac{1}{|J(\mathbf{x}_k(t),t)|}\delta(\mathbf{x}-\mathbf{x}_k(t)), \tag{25}$$

where the sum is taken over all zeros of $\mathbf{e}(\mathbf{x},t)$ on $\mathcal{P}^3$, having the coordinates $\mathbf{x}_k(t)$: $e^a(\mathbf{x}_k(t),t) = 0$, $a = 1,2,3$, $k = 1,\ldots,N$.

The form of this relation makes it possible to formally interpret it as the first Maxwell equation,

$$\operatorname{div}\mathsf{D}_q = 4\pi\rho_q, \tag{26}$$

for the electric field with induction D, with the components

$$\mathsf{D}_q^\alpha = \frac{|J|}{|\mathbf{e}|^3}e^a\frac{\partial x^\alpha}{\partial e^a}, \quad \alpha = 1,2,3, \tag{27}$$

where ρ_q is determined by Eq. (25) and has the form

$$\rho_q = e_0\sum_{k=1}^{N}\delta(\mathbf{x}-\mathbf{x}_k(t)). \tag{28}$$

The factor with the transformation Jacobian in the expression for ρ_q cancels. The meaning of such an association is, from a physical point of view, that flows of the field $\mathbf{e}$ through any closed surface σ surrounding the origin in the marker space $\mathcal{E}^3$ is equal to the flow of the field D_q through the image of $\mathcal{S}$ on $\mathcal{P}^3$ on the surfaces σ on $\mathcal{E}^3$ as a result of mapping $\mathbf{e}\to\mathbf{x}$. Since the flow of the $\mathbf{e}$ field through σ is equal to 1, the flow of the D_q field is also equal to 1. What is remarkable in this comparison is that discrete charges with a value of $q = 1$ appear in the theory, and at the same time they do not create problems with divergences of energy [2, 3]. It can be easily shown [2, 3] that the field D_q with the components (27) has a Coulomb asymptotic at the points $\mathbf{x}_k$, which form the carrier of the function $\delta(\mathbf{e})$, but the structure of $\mathcal{V}^3$ remains smooth.

Nevertheless, the use of D_q as induction of the fundamental electric field requires additional clarifications. First of all, it is necessary to introduce the concept of matter particles as some elements of the $\mathcal{V}^3$ structure. It is also necessary that two types of electric charges should appear in the theory, the positive and negative ones. Only then will there be a reliable basis for associating D_q with the induction field. To solve these problems, it is necessary to associate $e^a(\mathbf{x},t)$ with geometric properties of $\mathcal{V}^3$, which has not yet been done. We have only stated that $e^a(\mathbf{x},t)$ are markers of points of $\mathcal{V}^3$.

6. TOPOLOGICAL CELLS AND MATTER PARTICLES

To establish a connection of the properties of marker fields with the geometry of $\mathcal{V}^3$, we use the fact that this hypersurface $\mathcal{V}^3$ is allocated in $\mathcal{W}^4$ using Eq. (1) with some height function $\mathcal{F}(\mathbf{x},t)$, which we will further on call the *fundamental potential*. Based on the fact that $\mathcal{F}(\mathbf{x},t)$ determines the locus of the points of $\mathcal{V}^3$ in $\mathcal{W}^4$, this function itself must be a marker, i.e., it must be a function of the marker fields $e^a(\mathbf{x},t)$. It is useful to define the dependence type of $\mathcal{F} = \mathcal{F}(\mathbf{e})$ in a convenient way for using in particle theory. Such a simplest way is the relationship

$$\mathcal{F} = \mathcal{F}_i + \frac{\varepsilon_i}{2}|\mathbf{e}|^2, \tag{29}$$

written for each separate region of the space $\mathcal{V}_i$, $i = 1,2,\ldots$, bounded by the isosurface of the function $\mathcal{F}(\mathbf{x},t)$, inside which it reaches a single local extremum, a minimum or a maximum, or has no extremum at all. We will call such regions of space *simple topological cells*. The function $\mathcal{F}_0(t)$ is the value of the function $\mathcal{F}(\mathbf{x},t)$ at the extremum corresponding to $\mathcal{V}_i$, and the value of ε_i for a maximum is -1, and for a minimum $\varepsilon_i = 1$. Simple topological cells containing no extremum of the function $\mathcal{F}(\mathbf{x},t)$ will be called *empty* topological cells.

Any region of the space $\mathcal{P}^3$ bounded by a closed isosurface of $\mathcal{F}(\mathbf{x},t)$ will be called a topological cell. Besides simple topological cells, we select *basic topological cells* bounded by special isosurfaces on which there is at least one saddle point of the function

$\mathcal{F}(\mathbf{x}, t)$. A visual representation of what is meant by topological cells is given by Fig. 1, which shows an analogue of $\mathcal{V}^3$ in the form of a two-dimensional surface. The points P_1, P_2, P_3 are local extrema of the function $\mathcal{F}(x, y)$, which is an analogue of the fundamental potential $\mathcal{F}(\mathbf{x}, t)$. Extrema and special isolines of the function $\mathcal{F}(x, y)$ are projected onto the plane $\mathcal{P}^3$, bounding the basic topological cells. Simple topological cells are highlighted in light gray, and empty topological cells in dark gray.

In what follows, we will suppose that, from a topological point of view, $\mathcal{V}^3$ is arranged in a sufficiently simple way. We will assume that the number of critical points of the function $\mathcal{F}(\mathbf{x}, t)$ in any bounded region of space is finite, i.e., the function $\mathcal{F}(\mathbf{x}, t)$ as a hight function of a smooth hypersurface $\mathcal{V}^3$ in $\mathcal{W}^4$ is a Morse function [16]. This will simplify all necessary references to the topological properties of $\mathcal{V}^3$, which will be needed in the future. For example, in the framework of this general hypothesis, it can be assumed that the entire region of the hyperplane $\mathcal{P}^3$, on which the points of $\mathcal{V}^3$ are projected, can be completely divided into topological cells. This agreement is adopted in order not to consider, at least at the first stage of theory construction, too complex topological structures that are possible from a mathematical viewpoint.

The (29) relations unify the description of topological cells and make it possible to give an integer electric charge a topological meaning. According to (29), each simple topological cell has its own separate sheet $\mathcal{E}_i$ of the marker space. For nonempty simple cells, the origin on the Cartesian map of the marker space is mutually unambiguously mapped to the local extremum lying in this cell. At the same time, in accordance with (29), each isosurface $\mathcal{F}$ inside the cell is mapped into a concentric sphere in the Cartesian map of the sheet $\mathcal{E}_i$, whose radius is determined as

$$R = \sqrt{2|\mathcal{F} - \mathcal{F}_0|}. \tag{30}$$

The whole cell is then mapped to a ball centered by the origin. The radius of the ball to which the boundary $\partial\mathcal{V}_i$ of the cell $\mathcal{V}_i$ is mapped, is equal to

$$R_i = \sqrt{2|\mathcal{F}_i - \mathcal{F}_0|},$$

where $\mathcal{F}_i$ is the value of $\mathcal{F}(\mathbf{x}, t)$ on the isosurface that bounds the cell.

Since an empty cell does not contain extrema, the choice of $\mathcal{F}_0$ for it in Eqs. (29) can be arbitrary. Depending on the choice of $\mathcal{F}_0$ for an empty cell, it can be displayed in a spherical layer on the corresponding sheet $\mathcal{E}_i$. But with a special choice of the value of $\mathcal{F}_0$, coinciding with the value $\mathcal{F} = \mathcal{F}_s$ on one of the special isosurfaces $\mathcal{S}_i$ bounding it, the cell maps to a

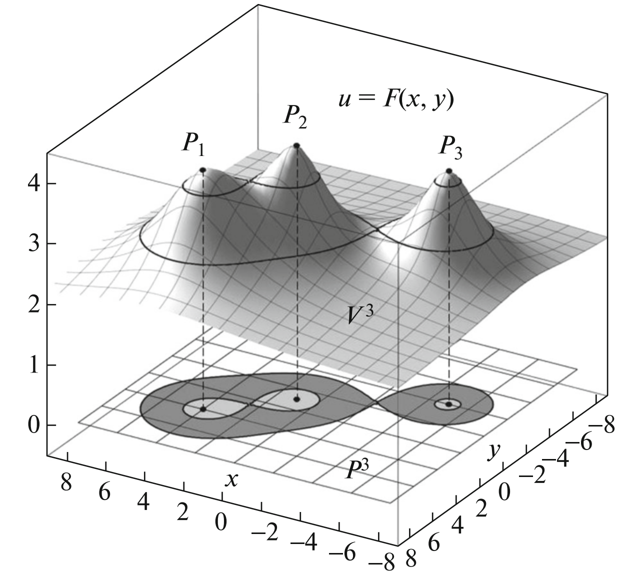


Fig. 1. Two-dimensional analogue of the physical hypersurface $\mathcal{V}^3$ with a visible presentation of all types of topological cells.

ball. In this case, the whole isosurface $\mathcal{S}_i$ maps to a point located at the origin of the sheet.

Nonsimple (complex) basic cells contain simple topological cells as structural elements, as can be seen in Fig. 1. The basic cells will be further interpreted as matter particles, whose structural elements will be simple topological cells. Let us note that in using such a principle in the description of matter particles, one can see a connection with the ideas that were expressed by Clifford in [13].

7. THE ELECTRIC CHARGE OF TOPOLOGICAL CELLS

The principle of matter particle presentation by basic topological cells must be confirmed by a proof that the dynamics of such objects obeys the laws of quantum theory and Newton's laws in a certain averaged meaning. Such proofs will be presented furter on. In addition to dynamic criteria, it is also necessary to indicate how the structure of particles is related to a discrete electric charge and other charge numbers such as the baryonic number. The general idea of such constructions was stated earlier in [1, 4, 5, 17, 18]. It makes sense to give a detailed presentation of this issue, related to an analysis of the topology of 3D regions of the hypersurface $\mathcal{V}^3$ in a separate paper, especially since not all details of the calculations for the charge properties of particles are clear by now. Here we will focus on a presentation of general principles of such constructions.

A starting point for further constructions will be the use of Eqs. (26), (27), and (28) as a method of the description of the fundamental electric field of particles, represented by topological cells. Based on the fact that, for each simple topological cell number i, the marker fields take values on a separate sheet $\mathcal{E}_i$, the relations (26), (27), and (28) should be written separately for each topological cell. In this case, each simple cell, due to the fact that Eq. (26) has in its r.h.s. a δ function at a point with the coordinates $\mathbf{x}_i(t)$, contains a point charge at this point. The sign of this charge should in essence be determined by where the electric induction or strength field is directed at the cell boundary—inward or outward. This is how the gradient field $\nabla\mathcal{F}$ behaves, its direction on the boundary of a simple cell is determined by the sign ε_i in Eq. (29). Using (29), we can determine where the D_q field is directed to $\partial\mathcal{V}_i$. Differentiating (29) in x^α, we find

$$\frac{\partial\mathcal{F}}{\partial x^\alpha} = \varepsilon_i e^a \frac{\partial e^a}{\partial x^\alpha}, \quad \alpha = 1, 2, 3.$$

Hence it follows

$$e^a \frac{\partial x^\alpha}{\partial e^a}\frac{\partial\mathcal{F}}{\partial x^\alpha} = \varepsilon_i |\mathbf{e}|^2. \tag{31}$$

The last relation shows that the field K with the components

$$\mathsf{K}^\alpha = |J| e^a \frac{\partial x^\alpha}{\partial e^a}, \tag{32}$$

which is a common element in expressions for both fields $\mathbf{g}$ and D_q, is transversal to the isosurfaces of the function $\mathcal{F}$, i.e., it is almost everywhere directed *outward* from the region bounded by the isosurface. Indeed, if $\varepsilon_i = -1$, then there is a maximum of $\mathcal{F}$ inside the simple cell, and the field $\nabla\mathcal{F}$ is directed inward. In this case, we have on the bounding isosurface $\partial\mathcal{V}_i$ that

$$e^a \frac{\partial x^\alpha}{\partial e^a}\frac{\partial\mathcal{F}}{\partial x^\alpha} = -|\mathbf{e}|^2 \leq 0,$$

that is, the projection of the field K onto $\nabla\mathcal{F}$ is negative everywhere on this boundary, except for critical saddle points. It just means that K is directed outward. Thus to take into account the sign of the cell charge in the expression for the induction, it is necessary to replace the field D_q on each simple but nonempty cell with a field of the following form:

$$\mathsf{D} = \varepsilon_i \mathsf{D}_q. \tag{33}$$

A similar relationship will take place for empty cells, in which the charge sign will be determined by the direction of the field $\nabla\mathcal{F}$, but it must be then consistent with charges in the neighboring simple cells. This issue is very important and will be discussed later. In this case, on each simple topological cell we have the first Maxwell equation for the discrete point charge:

$$\operatorname{div}\mathsf{D} = 4\pi\varepsilon_i\delta(\mathbf{x} - \mathbf{x}_i). \tag{34}$$

Having obtained a description of the electric field induction on each individual simple cell, it is now necessary to combine it into a unified field on the whole hyperplane $\mathcal{P}^3$. To do that, it is necessary to additionally require that for the D fields at cell boundaries, the boundary conditions standard for classical electrodynamics are valid. This issue was discussed in [2]. According to classical electrodynamics, the normal component of the induction field should experience a jump in values on the boundary of cells with conditional numbers 1 and 2, equal to the value of the surface charge density σ on the boundary:

$$\mathsf{D}_n^{(2)} - \mathsf{D}_n^{(1)} = 4\pi\sigma.$$

Since we are dealing with discrete charges, there cannot be any surface charge density on cell boundaries, $\sigma = 0$. Therefore, the condition for D should turn into its continuity condition on cell boundaries: $\mathsf{D}_n^{(2)} = \mathsf{D}_n^{(1)}$. This condition, by using (27), (33) with (31), takes the form

$$\left.\frac{|J|}{|\mathbf{e}|}\right|_1 = \left.\frac{|J|}{|\mathbf{e}|}\right|_2. \tag{35}$$

It has been taken into account here that the function $\mathcal{F}$ itself and its derivatives are everywhere continuous in $\mathcal{P}^3$. We will discuss the meaning of these conditions later, when revealing in more detail the meaning of the function $|J|$ in particle dynamics.

8. THE ELECTRIC CHARGE AND EULER CHARACTERISTIC OF TOPOLOGICAL CELLS

To complete the description of the electric induction field D and its "resources," it is necessary to specify a general method of calculating the charge of complex topological cells, which, as already mentioned, will be further considered as matter particles. In physics, the magnitude of the charge contained in a certain region of space $\mathcal{V}$ is associated with the flux of the induction field through the boundary $\in \mathcal{V}$ of this region:

$$q = \oint\limits_{\partial\mathcal{V}} \mathsf{D}^\alpha d\sigma_\alpha.$$

For the field D, an expression for the charge contained in a region bounded by any isosurface of the fundamental potential $\mathcal{F}$ inside the corresponding cell $\mathcal{V}_i$, has the form

$$q_i = \oint\limits_{\partial\mathcal{V}_i} \mathsf{D}^\alpha d\sigma_\alpha = \oint\limits_{\partial\mathcal{V}_i} \frac{\varepsilon_i |J|}{|\mathbf{e}|^3} e^a \frac{\partial x^\alpha}{\partial e^a} d\sigma_\alpha = \varepsilon_i. \tag{36}$$

This expression, according to [24], represents the Euler characteristic of the region $\mathcal{V}_i$ for which the

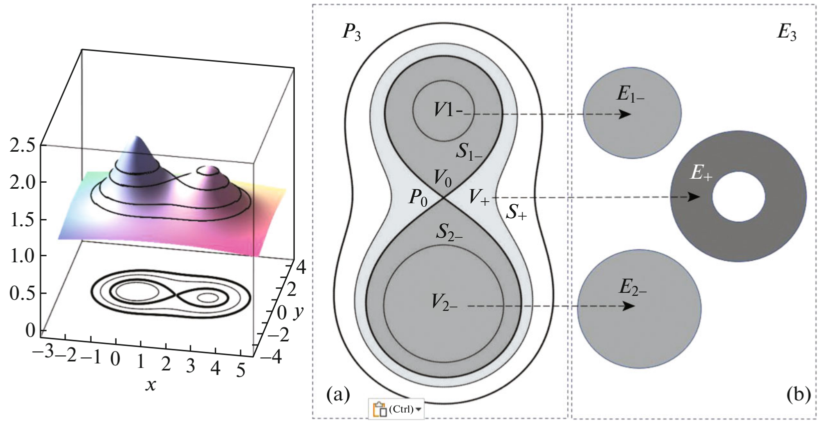


Fig. 2. Correspondence of 2D analogues of topological cells on $\mathcal{P}^3$ to sheets in the marker space $\mathcal{E}^3$. P_0 is a saddle point of the function $\mathcal{F}$.

charge is calculated. The Euler characteristic [19, 20, 25–27] is a topological invariant of the corresponding region, it does not change at smooth deformations of this region [19, 20, 25–27]. Thus the discrete electric charge in this theory [1, 2] turns out to be associated with an integer quantity, the Euler characteristic of topological cells:

$$q_i = \varepsilon_i \chi(\mathcal{V}_i). \tag{37}$$

The sign of the particle charge in this relation as the field flux magnitude is determined by the direction of the field **D** at the cell boundary. This direction is determined by the number ε_i, which is called in topology the Poincaré-Hopf index [25, 26, 27]. The above relation is an important element of the new theory, which allows us to construct a charge classification of possible structures of topological cells [1, 4, 5]. The most significant in Eq. (37) is that the particle charge turns out to be a property of the topology of the material hypersurface itself $\mathcal{V}^3$. This means that electrodynamics also receives a geometric and topological interpretation. Such an attempt was made earlier by Wheeler and Misner in the framework of GR [21–23].

For clearer understanding of how the relations (36) work, let us make some clarifications. The first one is that the Euler characteristic $\chi(\mathcal{V}_i)$ of a region $\mathcal{V}_i$ in 3D space, bounded by a closed surface $\partial\mathcal{V}_i$, is equal to half of the Euler characteristic of this boundary:

$$\chi(\mathcal{V}_i) = \frac{1}{2}\chi(\partial\mathcal{V}_i). \tag{38}$$

This means that the electric charge of particles is determined by the topology of the boundary of topological cells, which simplifies the classification of possible particle structures, which was described earlier in [1, 2, 5]. The main ideas of such a classification will be presented here later on.

Another clarification concerns the calculation of the Euler characteristic, and consequently particles charges, based on the boundary structure of the corresponding topological cells. By definition, basic topological cells are bounded by special isosurfaces containing saddle points of the function $\mathcal{F}$. Therefore, special isosurfaces are not smooth everywhere. Being isosurfaces of smooth functions, they nevertheless have geometric singularities precisely at the saddle points $\mathcal{F}(\mathbf{x}, t)$. Therefore, the calculation of the Eulerian characteristic of such isosurfaces has its own peculiarities. However, difficulties with calculating particle charges can be avoided by introducing two auxiliary isosurfaces constructed for each special isosurface according to the following principle. Suppose that the special isosurface $\partial\mathcal{V}_i$ corresponds to the value $\mathcal{F}_0$. Then the auxiliary isosurfaces $\mathcal{S}_{i+}$ and $\mathcal{S}_{i-}$ will be understood as isosurfaces corresponding to the values of $\mathcal{F}$, equal to $\mathcal{F}_0 + \delta\mathcal{F}$ and $\mathcal{F}_0 - \delta\mathcal{F}$, respectively, where $\delta\mathcal{F}$ is an infinitesimal quantity whose sign is defined as to fulfill the conditions $\mathcal{V}_{i-} \in \mathcal{V}_i \in \mathcal{V}_{i+}$. This means that $\mathcal{V}_{i-}$ with the boundary $\mathcal{S}_-$ lies inside the region $\mathcal{V}_i$ bounded by the special isosurface. In turn, $\mathcal{V}_i$ itself is located inside the region $\mathcal{V}_{i+}$. It is important that the isosurfaces $\mathcal{V}_{i\pm}$ are closed smooth 2D surfaces whose Euler characteristic can be calculated without difficulty. On the other hand, for an external observer, the charge sign of a particle, which is by definition a basic topological cell, will be a flow through the external isosurface $\mathcal{V}_{i+}$, which simplifies a calculation of particle charges and their classification according to the structure of smooth isosurfaces $\mathcal{V}_{i+}$.

Figure 2 presents a two-dimensional analogue of the structure of two simple cells bounded by a special isosurface containing a single saddle point P_0 of the function $\mathcal{F}$. This structure is similar to the cells generated by two extrema of P_1 and P_2 of the functions $\mathcal{F}$, shown in Fig. 1. Figure 2 shows the cells themselves,

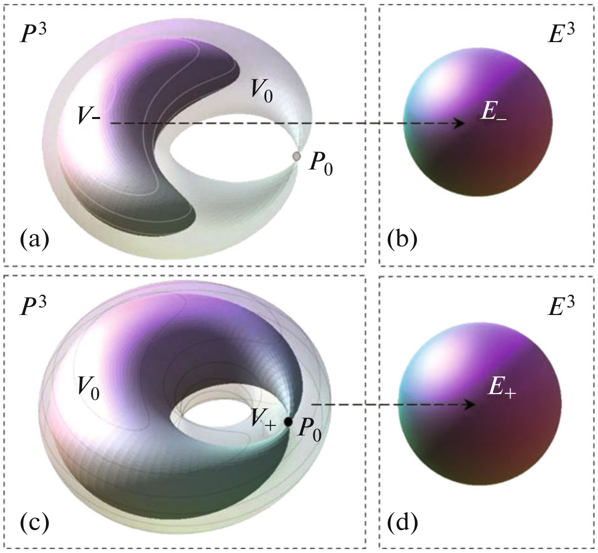


Fig. 3. Correspondence of the auxiliary isosurfaces $\mathcal{V}_-$ and $\mathcal{V}_+$ to the special isosurface $\mathcal{V}_0$ with a single saddle point P_0 of the function $\mathcal{F}$.

located on $\mathcal{P}^3$, and, on the right, the corresponding sheets of the marker space. The special isosurface of the function $\mathcal{F}$ is the inner boundary of the auxiliary cell $\mathcal{V}_+$ and the outer boundary is the isoline $\mathcal{S}_+$. The curves $\mathcal{S}_{1-}$ and $\mathcal{S}_{2-}$ are diffeomorphic to circles, and the cells $\mathcal{V}_{1-}$ and $\mathcal{V}_{2-}$ are circles. In the marker space, the isolines $\mathcal{S}_{1-}$ and $\mathcal{S}_{2-}$ also correspond to circles, which means that the flows of $\mathbf{D}$ through both of these isolines are equal to $+1$ if they contain minima of $\mathcal{F}$, and are equal to -1 if there are maxima. These extrema correspond to point charges. The outer boundary of $\mathcal{V}_+$ is also a curve diffeomorphic to a circle. In the marker space, a curve that corresponds to it is also a circle. Therefore, the flow through this isoline is either $+1$ or -1. Therefore, at a transition through a special isosurface, the charge value decreases by 1. Since the field $\mathbf{D}$, in accordance with the conditions (35), is continuous on $\mathcal{S}_+$, with the exception of the saddle point P_0 of the function $\mathcal{F}$, it is necessary to assign to this point a charge inverse to the charge of the cells $\mathcal{V}_{1-}$ and $\mathcal{V}_{2-}$. This is confirmed by the fact that the field $\mathbf{D}$ has a Coulomb asymptotics (see [2]). Thus in this theory an electric charge should be attributed to all critical points of the fundamental potential, including its saddle points.

The 2D analogue considered in Fig. 2 as an example, is useful for illustrating why an electric charge must be attributed to saddle points of $\mathcal{F}$. However, it does not give a complete understanding of how works the relation (37), connecting the electric charge with the Euler characteristic of a topological cells.

For a clearer explanation of this relationship, consider the example shown in Fig. 3. In this figure, a and c, an image of the basic cell $\mathcal{V}_0$ is presented in a semitransparent form, being bounded by a special isosurface with one saddle point P_0 of the function $\mathcal{F}$. The auxiliary topological cell $\mathcal{V}_-$, lying inside $\mathcal{V}_0$, has the boundary $\partial\mathcal{V}_-$ diffeomorphic to a sphere. This region is mapped to a ball in the marker space, shown in panel b. Panel c shows in real color the same basic cell $\mathcal{V}_0$. The auxiliary cell $\mathcal{V}_+$, having a boundary in the form of a torus, is shown in a semi-transparent form. The image of this cell after mapping $\mathbf{x} \to \mathbf{e}$ to the marker space $\mathcal{E}^3$ is a spherical layer, i.e., a part of the ball, depicted in Panel d. The total flow through the boundary of the cell $\mathcal{V}_-$ is equal to $+1$ or -1, depending on the type of extremum of $\mathcal{F}$ lying in $\mathcal{V}_-$. Since the boundary of $\mathcal{V}_+$ is diffeomorphic to a torus and has a sphere in its image on $\mathcal{E}^3$, the flow through $\partial\mathcal{V}_+$ is equal to the Euler characteristic of the torus, which, as is well known, is zero [19, 20, 25, 27]. This is confirmed by the *general rule: the saddle points $\mathcal{F}$ lying on special isosurfaces should be assigned a charge inverse to the charge in $\mathcal{V}_-$, and the total charge of a particle being a basic topological cell, should be calculated as the Euler characteristic of the auxiliary region $\mathcal{V}_+$ (37) or half of the Euler characteristic of its boundary (38).*

9. BASICS OF THE CHARGE CLASSIFICATION OF PARTICLES

Following the general rule for calculating the charge of particles identified with basic topological cells, it is possible to construct a simple charge classification of particle types by the Euler characteristics of the outer boundary of the auxiliary cell, $\mathcal{V}_+$ [1–5]. Since, as a postulate in this paper, it is assumed that the function $\mathcal{F}(\mathbf{x}, t)$ is smooth, almost all its nonsingular isosurfaces are 2D closed orientable manifolds. The classification of such isosurfaces reduces to a theorem [19, 20, 25–27], according to which any such isosurface $\mathcal{S}_g$ is diffeomorphic to a sphere with g handles. The Euler characteristic of all such isosurfaces is

$$\chi(\mathcal{S}_g) = 2(1-g). \tag{39}$$

The Euler characteristic of a sphere is $\chi(\mathcal{S}_0) = 2$, for a torus $\chi(\mathcal{S}_1) = 0$, etc. In agreement with (38), the Euler characteristics of regions $\mathcal{V}_g$, having as boundaries $\mathcal{S}_g = \partial\mathcal{V}_g$, and consequently the particle charges are equal to

$$q = \varepsilon\chi(\mathcal{V}_g) = \varepsilon\frac{1}{2}\chi(\mathcal{S}_g) = 1 - g, \tag{40}$$

respectively. A sequence of such particles is depicted in Fig. 4.

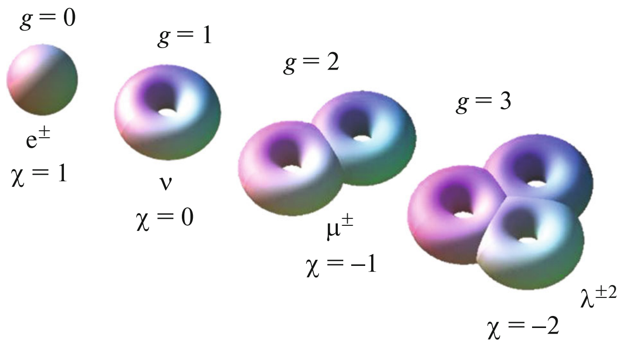


Fig. 4. The structure of topological cells depending on their Euler characteristics.

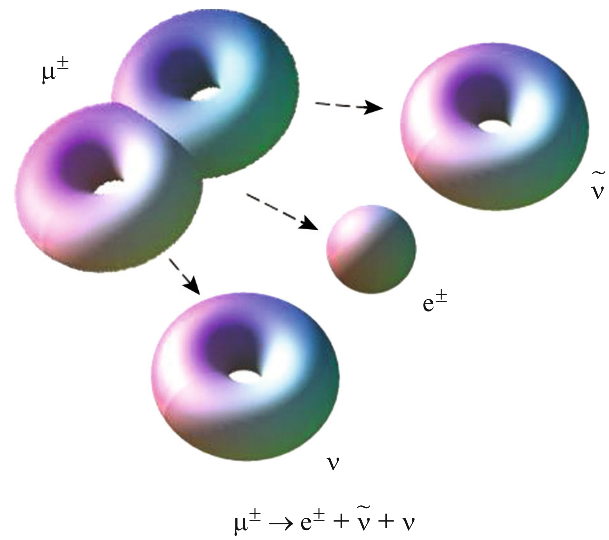


Fig. 5. Topological reconstruction of muon decay: $\mu^+ \to e^+ + \nu + \tilde{\nu}$.

9.1. Interpretation of the Leptons

The simplest interpretation of particles whose charges are calculated by Eq. (40) and are shown n Fig. 4, is their interpretation as leptons according to Table 1.

An indirect confirmation of this interpretation is the muon decay, whose topological reconstruction is shown in Fig. 5. The muon corresponds to a topological cell with $g = 2$. The main channel of muon decay is the decay into two neutrinos and an electron, which corresponds to Fig. 5. The particle designated in the table as $\lambda^{\mp 2}$ is a hypothetic lepton with a charge ∓ 2, whose decay, according to the topological reconstruction, should occur into a muon, an electron and a neutrino, or into two electrons and three neutrinos.

The classification corresponding to Table 1 obviously does not restrict all possible structures of topological cells. The simplest addition is to include structures whose auxiliary isosurface is fixed, but the number of saddle points of $\mathcal{F}$ that lie on a special isosurface changes. This is shown in Fig. 6 using as an example a toroidal surface $\mathcal{V}_+$, depicted in translucent color with one, two and three saddle points P_1, P_2, P_3. Since the charge of such cells remains zero, they can be compared with different types of neutrinos, which in Table 1 and in Fig. 4 correspond to a torus with $g = 1$.

Table 1. Classification of leptons

g	0	1	2	3
$\chi(\partial\mathcal{V}_+)$	2	0	-2	-4
q	±1	0	∓1	∓2
Type	e^+	ν	$\mu^{\mp}$	$\lambda^{\mp2}$

9.2. Interpretation of the Structure of Nucleons

In addition to these simple types of cells, other possible topological structures should be included in the general classification. Such structures primarily include the Wheeler-Misner [21–23] topological handles, using which the authors of this idea proposed to explain the electric charge in the framework of GR, which would avoid the energy divergences for a point charge that are standard for electrodynamics. The Wheeler-Misner topological handles can be an element of topologies of smooth 3D hypersurfaces [27] and therefore should be included in the general classification of particles. As was shown in [5], the most natural way to include Wheeler handles in the general scheme is to establish a relationship between their number and the baryon charge.

Figure 7 presents 2D analogues of Wheeler handles. The handles are oriented on different sides of $\mathcal{V}^3$. For simple reasons, each handle should be assigned a baryon charge $b = +1$ or $b = -1$, depending on which side of $\mathcal{V}^3$ it is "glued" into this hypersurface.

As already discussed, the electric charge of such structures will be entirely determined by the external auxiliary isosurface of the $\mathcal{V}_+$ of the function $\mathcal{F}$. This outer isosurface can be either a sphere, or a torus, or another smooth isosurface. For clarity, in Fig. 8 the Wheeler handle is presented with some elements of internal structure determining its charge properties. In the 3D case, the handle structure will contain an extremum and special isosurfaces $\mathcal{V}_1$ and $\mathcal{V}_2$. Each of $\mathcal{V}_1$ and $\mathcal{V}_2$ is a pair of spheres "glued" together at one point. The outer isosurface, whose analogue is $\mathcal{V}_+$, can be a sphere or a torus. In these cases, it is quite natural to assume that a handle with $\mathcal{V}^+$ in the

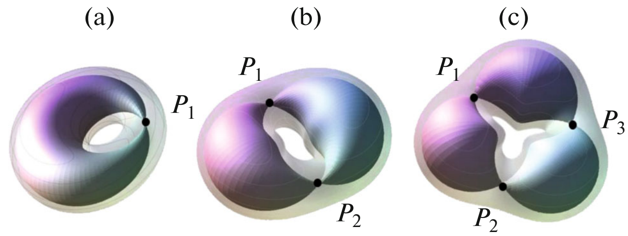


Fig. 6. Possiblee structures of different types of neutrino.

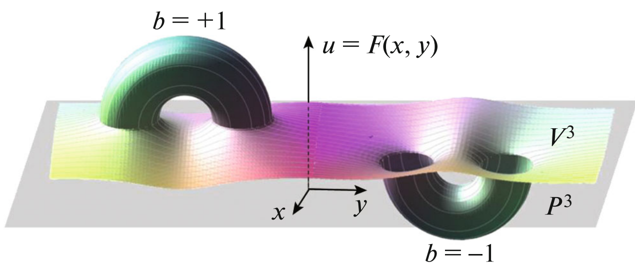


Fig. 7. 2D analogue of Wheeler handles with interpretation of the baryonic charge.

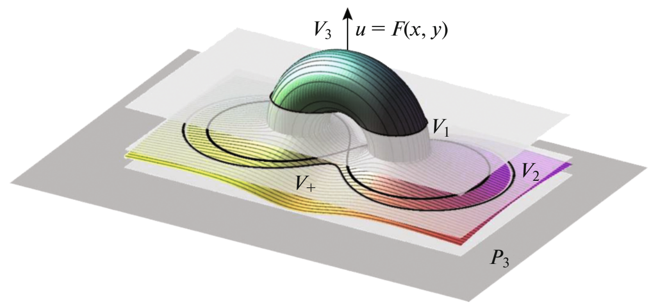
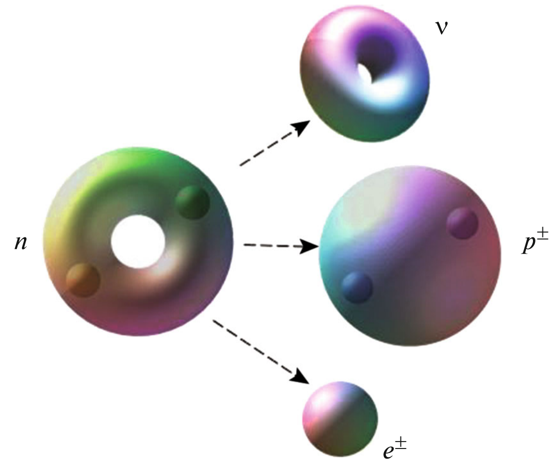


Fig. 8. Structure elements of a Wheeler handle.



$n \to e^{\pm} + p^{\pm} + \nu$

Fig. 9. Interpretation of nucleons and a topological reconstruction of neutron decay.

form of a sphere is a proton model p^+, and with $\mathcal{V}^+$ in the form of a torus it is a neutron model with zero charge. The adequacy of this interpretation can be seen in the corresponding topological reconstruction of the decay of a free neutron, presented in Fig. 9. This figure shows a neutron and a proton in a form that an external observer would have to see. Inside the translucent outer isosurfaces in the form of a torus (neutron) and a sphere (proton), two spherical objects can be seen, which conventionally represent the necks of Wheeler handles. The decay of a free neutron occurs according to the weak decay scheme, resulting in the emergence of a proton (antiproton), an electron (positron) and a neutrino. This reconstruction, as well as the reconstruction of the muon decay (Fig. 5), suggests that from the point of view of the geometry of space, a weak decay is a topological rearrangement of the outer isosurface of the particle.

In this paper we will not dwell on the development of a general scheme of interpretation of all types of particles, including mesons and other types of baryons, as well as issues related to unitary symmetries, which are the basis of modern understanding of the particle structure. In part, a description of some mesons and nuclei of simple atoms which can be established based solely on their charge properties, is given in [5]. Here we just note that the charge of almost all types of particles can be described by a general formula that follows from the general formula for calculating the Euler characteristic in terms of the Betti numbers p_i, $i = 0, \ldots, 3$ [19, 20, 24, 25, 27]:

$$\chi(\mathcal{V}) = p_0 - p_1 + p_2 - p_3. \tag{41}$$

The Betti numbers of 3D manifolds with an edge are equal to [19, 20, 24, 27]

$$p_0 = 1, \quad p_1 = g + b, \quad p_2 = 1 + b, \quad p_3 = 1,$$

where g is the number of handles in the structure of the cell boundary (determined the charge), while b is the number of Wheeler handles, i.e., the particle's baryonic number. As a result, the relation (41) may be interpreted as the Gell-Mann-Nishijima formula [28]:

$$Q = \frac{b + S}{2} + J_3,$$

where Q is the particle charge, S is the strangeness, and J_3 is the isospin projection. The numbers S and J_3 can be explicitly calculated from the values of the Betti numbers [5]. The issue of a topological classification of particles, which is important in all respects, requires using not only the charge properties of particles in their interpretation, but also calculations of their mass and spin. It makes sense to carry out such calculations in a separate paper. Note only that the topological interpretation of the Gell-Mann-Nishijima formula indicates that in this theory there is no need to introduce the concept of quarks as particles with a fractional electric charge, which are used in chromodynamics to explain the existence of unitary symmetries in the particle structure.

10. FUNDAMENTAL ELECTROMAGNETIC AND GENERALIZED GRAVITATIONAL FIELDS

Let us now consider how the fundamental magnetic field emerges in this theory and how it is related to marker fields. To do so, consider how the field $\mathbf{K}$ (32) changes with time due to marker transferin accordance with (17). All necessary constructions have been described in detail in [7]. Therefore, here we present their result, making reference to the details of these constructions presented in Appendix A.

The initial problem to be solved next is to calculate the time derivative of the $\mathbf{K}$ field. To do that, we present this field using the identities (A.1) in the following form:

$$\mathbf{K} = \varepsilon_{abc}[\nabla e^b \times \nabla e^c]e^a. \tag{42}$$

Here ε_{abc} is the antisymmetric Levi-Civita symbol. This form of $\mathbf{K}$ follows from a component-by-component usage of the inverse Jacobi matrix of the mapping $\mathbf{e} \to \mathbf{x}$. Now calculate the time derivative of the $\mathbf{K}$ field using (42):

$$\frac{\partial \mathbf{K}}{\partial t} = \varepsilon_{abc}[\nabla e^b \times \nabla e^c]\frac{\partial e^a}{\partial t} + 2\varepsilon_{abc}\left[\nabla \frac{\partial e^b}{\partial t} \times \nabla e^c\right] e^a. \tag{43}$$

Let us consider separately the two terms in the r.h.s. of this relation. To calculate them, we use the identities given in Appendix A and the transfer equation (17) for marker fields. Then we have

$$\varepsilon_{abc}[\nabla e^b \times \nabla e^c]^\alpha \frac{\partial e^a}{\partial t} = -\varepsilon_{abc}[\nabla e^b \times \nabla e^c]^\alpha \frac{\partial e^a}{\partial x^\beta}\mathsf{V}^\beta = -|J|\mathsf{V}^\alpha. \tag{44}$$

Here we have used the identity (A.2). As a result, for the second term in (43) we find:

$$\varepsilon_{abc}\left[\nabla \frac{\partial e^b}{\partial t} \times \nabla e^c\right] e^a = \varepsilon_{abc}\varepsilon^{\alpha\beta\gamma}\frac{\partial}{\partial x^\beta}\left[e^a \frac{\partial e^b}{\partial t}\frac{\partial e^c}{\partial x^\gamma}\right] - \varepsilon_{abc}\varepsilon^{\alpha\beta\gamma}\frac{\partial e^a}{\partial x^\beta}\frac{\partial e^b}{\partial t}\frac{\partial e^c}{\partial x^\gamma} = \operatorname{curl} \mathbf{Z} - |J|\mathbf{V}. \tag{45}$$

The field $\mathbf{Z}$ has the components

$$\mathsf{Z}_\gamma = \varepsilon_{abc}e^a \frac{\partial e^b}{\partial t}\frac{\partial e^c}{\partial x^\gamma} = -\varepsilon_{abc}e^a\mathsf{V}^\beta \frac{\partial e^b}{\partial x^\beta}\frac{\partial e^c}{\partial x^\gamma} = -\mathsf{V}^\beta\lambda_{\beta\gamma}. \tag{46}$$

Uning now the indentities (A.3) and (A.4), we arrive at the relations

$$\lambda_{\mu\nu} = \frac{1}{2}\mathsf{K}^\gamma\varepsilon_{\gamma\mu\nu}. \tag{47}$$

Substituting this into (46), we finally have

$$\mathsf{Z}_\gamma = -\mathsf{V}^\beta\lambda_{\beta\gamma} = \frac{1}{2}\mathsf{V}^\beta\mathsf{K}^\mu\varepsilon_{\mu\beta\gamma}, \tag{48}$$

or, in the vector form,

$$\mathbf{Z} = -\frac{1}{2}[\mathbf{K} \times \mathbf{V}]. \tag{49}$$

Substituting the resulting relations (44), (45) and (49) to (43), we finally obtain

$$\frac{\partial \mathbf{K}}{\partial t} = -\operatorname{curl}\Big([\mathbf{K} \times \mathbf{V}]\Big) - 3|J|\mathbf{V}. \tag{50}$$

It is the sought-for induction equation for the field $\mathbf{K}$.

Let us now use the fact that the fields $\mathbf{D}$ and $\mathbf{g}$ differ from $\mathbf{K}$ by only functional factors:

$$\mathbf{D} = \frac{\varepsilon}{|\mathbf{e}|^3}\mathbf{K}, \quad \mathbf{g} = \frac{4\pi G m_0}{3}\mathcal{M}(\mathbf{e})\mathbf{K}.$$

Accordingly, the induction equations for these fields will have the form [7]

$$\frac{\partial \mathbf{D}}{\partial t} = -\operatorname{curl}\Big([\mathbf{D} \times \mathbf{V}]\Big) - 4\pi\rho_e\mathbf{V}, \tag{51}$$

$$\frac{\partial \mathbf{g}}{\partial t} = -\operatorname{curl}\Big([\mathbf{g} \times \mathbf{V}]\Big) - 4\pi G\rho_m\mathcal{R}\mathbf{V}, \tag{52}$$

where $\rho_m = m_0\mathcal{M}(\mathbf{e})|J|$ is the mass density of space, the function $\mathcal{R}$ is defined by (30), and

$$\rho_e = \sum_{k=0}^{N}\varepsilon_k\delta(\mathbf{x} - \mathbf{x}_k(t)).$$

In the last expression, the sum is taken over all critical points of the fundamental potential $\mathcal{F}$, including its saddle points. Comparing (51) with the corresponding fourth equation of Maxwell's theory, we establish that in the present theory the fundamental magnetic field strength $\mathbf{H}$ should be calculated as

$$\mathbf{H} = \frac{1}{c}[\mathbf{D} \times \mathbf{V}] + \nabla\Phi_H, \tag{53}$$

where c is the speed of light, and Φ_H is a scalar potential of the magnetic field which should satisfy the condition that magnetic charges are absent (Maxwell's second equation),

$$\operatorname{div}\mathbf{H} = \frac{1}{c}\operatorname{div}[\mathbf{D} \times \mathbf{V}] + \Delta\Phi_H = 0.$$

Accordingly, the electric current density will be calculated by the formula

$$\mathbf{j} = \frac{4\pi}{c}\rho_e \mathbf{V},$$

i.e., it will coincide with the current density of point charges. Then, automatically, the electric charge conservation law will hold:

$$\frac{\partial}{\partial t}\rho_e + \operatorname{div}\mathbf{j} = 0.$$

The gravitational field induction equation (52) is absent in the classical theory of gravity, though its formal existence was discussed in [12]. The absence of a necessity to have such a field in classical theory, despite the obvious similarity of the theories of gravity and electromagnetism, follows from the fact that in classical celestial mechanics and astrophysics it did not make sense to include into consideration any additional fields at the achievable accuracy of measurements. The situation has changed only in the recent decades, when the accuracy of measurements has reached such a level that using atomic clocks and radars it is possible to measure effects that are usually attributed to the effects of SR and GR. In GR, the gravitational field is not reduced to a single gradient strength field (free fall acceleration) and is described by very complex Einstein equations for six independent components of the metric tensor.

In the theory considered here, according to (52), it is necessary to introduce the field $\mathbf{Z}$, which can be called, as in [12], a gravimagnetic field, and which should have the form

$$\mathbf{Z} = \frac{1}{c}[\mathbf{g} \times \mathbf{V}] + \nabla\Phi_G, \tag{54}$$

where Φ_G is a potential playing the same role as Φ_H for the magnetic field. The field $\mathbf{j}_G$,

$$\mathbf{j}_G = \rho_m \mathcal{R}\mathbf{V},$$

should be considered as the mass current density. The factor c^{-1} in the expression (54) is introduced by analogy with (53), reflecting the relative smallness of the gravimagnetic field effect on the dynamics of matter particles, like the magnetic field in Maxwell's theory. In Maxwell's theory, the factor c^{-1} gets into the dynamic equations for charged particles moving at a speed of $\mathbf{v}$ in the expression for the Lorentz force:

$$\mathbf{F}_L = -\frac{q}{c}[\mathbf{H} \times \mathbf{v}]. \tag{55}$$

The Lorentz force expresses the experimental data on the motion of charged particles in a magnetic field. So far there are no serious grounds to believe that an effect of the gravimagnetic field on particles should be much different from the effect of the Lorentz force. The similarity of induction equations for $\mathbf{D}$ and $\mathbf{g}$ suggests this idea. As a result, it should be assumed that in reality the gravimagnetic force acts on particles by a similar rule:

$$\mathbf{F}_G = -\frac{m}{c}[\mathbf{Z} \times \mathbf{v}]. \tag{56}$$

The factor c^{-1} apparently determines the smallness of the effects detected in the experiments of recent decades and attributed to GR. To confirm this hypothesis, it is necessary to carry out calculations of the dynamics of bodies in celestial mechanics using (56) and thus to confirm or refute it.

11. THE FIELD ENERGY AND MASS

To complete the description of the dynamics of the fundamental fields connected with points of the hypersurface $\mathcal{V}^3$ by numbered marker fields, it is necessary to introduce field energy into the theory. Note that the mass density of the hypersurface $\mathcal{V}^3$ in a projection onto $\mathcal{P}^3$ was introduced by Eqs. (18). It is logical to assume that the field energy should be a conserved quantity, and its density should in some way repeat at least formally the general form of the energy density of the electromagnetic field.

The standard form of the electric field density in Maxwell's electrodynamics is, up to a multiplier, a scalar product of the induction vectors $\mathbf{D}$ and the electric field strength $\mathbf{E}$: $W = (\mathbf{E}, \mathbf{D})/4\pi$. In the present theory, the fundamental electromagnetic field strength is not yet present.

We introduce the strength $\mathbf{E}$ of the fundamental field so that its energy, calculated by the formula

$$\mathcal{W} = \frac{1}{4\pi}\int\limits_{\mathcal{V}} (\mathbf{D}, \mathbf{E})\, dV, \tag{57}$$

is a conserved quantity. To achieve that, it is sufficient to suppose [1–3] that the strength $\mathbf{E}$ has the following general form:

$$\mathbf{E} = Q(\mathbf{e})\nabla\mathcal{F}, \tag{58}$$

where $Q(\mathbf{e})$ is some function of the markers. Then, on each simple topological cell $\mathcal{V}_i$, with (31), we have

$$(\mathbf{D}, \mathbf{E}) = \frac{|J| Q(\mathbf{e})\varepsilon_i}{|\mathbf{e}|^3} e^a \frac{\partial x^\alpha}{\partial e^a}\frac{\partial \mathcal{F}}{\partial x^\alpha} = \frac{|J| Q(\mathbf{e})}{|\mathbf{e}|}.$$

It follows from this relation that if we choose $Q(\mathbf{e})$ as

$$Q(\mathbf{e}) = 4\pi k \mathcal{M}(\mathbf{e})|\mathbf{e}|, \tag{59}$$

then the energy density $\mathcal{W}$ will have the form

$$\rho_W = 4\pi k \mathcal{M}(\mathbf{e})|J|, \tag{60}$$

and the field energy will be connected with the mass according to

$$\mathcal{W} = k \int\limits_{\mathcal{V}_i} \mathcal{M}(\mathbf{e})|J|dV. \tag{61}$$

If we now choose $k = m_0c^2$, where m_0 is a fundamental mass constant, and c is the speed of light, then the latter relation becomes the Einstein formula that connects the mass and energy of matter,

$$\mathcal{W} = Mc^2.$$

As a result, one of the most important relations of SR arises in the present theory without aneed to introduce the postulates of SR. It should be noted that any choice of $Q(\mathbf{e})$ will make the corresponding quantity $\mathcal{W}$ an integral of motion, since $\mathcal{M}(\mathbf{e})|J|$ is a conserved density. Therefore, the choice of (59) should be distinguished in a special way among all other choices of $Q(\mathbf{e})$. Indeed, substituting (59) into (58), we find:

$$\mathsf{E} = 4\pi\mathcal{M}(\mathbf{e})|\mathbf{e}|\nabla\mathcal{F} = \varepsilon_i\mathcal{M}(\mathbf{e})\nabla\Phi_E,$$

where

$$\Phi_E = \frac{4\pi}{3}|\mathbf{e}|^3.$$

It follows that Φ_E is the volume of a ball on the Cartesian map of the marker space, with a center at the origin and a radius $|\mathbf{e}| = R = \sqrt{2(\mathcal{F} - \mathcal{F}_i)}$, i.e., the actual number of markers inside the special isosurface $\mathcal{F}$ corresponding to $R(\mathbf{x}, t)$. It should be noted that there is no simple connection between the induction field D and E in this theory, which indicates an anisotropy of the electrical properties of the "curved" hypersurface $\mathcal{V}^3$ in the interpretation of classical electrodynamics of continuous media.

12. PARTICLE DYNAMICS AND NEWTON'S EQUATIONS

Having obtained a general description of the fundamental gravitational and electromagnetic fields as properties of the hypersurface $\mathcal{V}^3$, it is now necessary to describe the dynamics of particles, which are considered in this theory as extended (nonlocal) objects, but with point singularities coinciding with critical points of the fundamental potential $\mathcal{F}(\mathbf{x}, t)$. Such particle structure in this theory allows us to find a way to interpret the duality of quantum particles. In quantum theory, particles behave in some conditions as nonlocal objects, waves, and in others as point objects. This duality, or even contradiction, belongs to the basic postulates of quantum theory. In particular, according to Born's statistical postulate [10, 29], the probability density ρ_p to find a particle at a point with coordinates $\mathbf{x}$ at time t is equal to the absolute value of the wave function $\Psi(\mathbf{x}, t)$:

$$\rho_p = |\Psi(\mathbf{x}, t)|^2. \tag{62}$$

Thus, the idea of a poin nature of particles is introduced into quantum theory, and on the other hand, the whole description is based on nonlocal mathematical structures associated with the function $\Psi(\mathbf{x}, t)$, describing the wave properties of particles. As is well known (see, e.g., [10]), this leads to the impossibility of obtaining a rational interpretation of the entire set of laws of quantum theory, which still gives remarkable results when calculating many effects of matter structure at the level of atoms and molecules.

To build a new interpretation of quantum theory in the framework of the proposed new theory, it is possible to approach the solution of this problem from several sides. One of the ideas is to construct the dynamics of the critical points of the function $\mathcal{F}(\mathbf{x}, t)$ with known dynamics of $\mathcal{F}$ itself. This approach is possible and provides some useful information on particle dynamics, but it is hard to connect it with the parameters of particle motion measured in the experiment. In reality, we do not directly observe the extrema themselves. An approach that can be compared to the approach of modern quantum theory, consists in constructing an averaged dynamics of particles as extended objects. To implement such an idea, it is above all necessary to find a suitable replacement of Born's statistical postulate (62).

To obtain the averaged characteristics of particles as extended objects, we have at least one conserved density $|J|$ at our disposal. Therefore, for each particle that corresponds to some cell $\mathcal{V}_i$ bounded by a special closed isosurface, using $\rho = \mathcal{M}(\mathbf{e})|J|$ one can assign the average coordinates $\mathbf{X}_i(t)$ [1, 3, 4] by the following rule:

$$X_i^\alpha = \frac{m_0}{M} \int\limits_{\mathcal{V}_i} x^\alpha \rho d\mathcal{V}, \tag{63}$$

where $M_i = m_0 \int_{\mathcal{V}_i} \rho d\mathcal{V}$ is a normalizing factor coinciding with the particle mass if the mass of the whole set of points of $\mathcal{V}_i$ is assigned to the particle, and the mass density is taken as $\rho_m = m_0\mathcal{M}(\mathbf{e})|J|$. Differentiating Eq. (63) in t, we arrive at the relations

$$\begin{aligned} V_i^\alpha = \frac{dX^\alpha}{dt} &= \frac{m_0}{M_i} \int\limits_{\mathcal{V}_i} x^\alpha \frac{\partial |J|}{\partial t} d\mathcal{V} + \oint\limits_{\partial\mathcal{V}_i} |J| x^\alpha v^\beta d\sigma_\beta \\ &= \frac{m_0}{M_i} \int\limits_{\mathcal{V}_i} \mathsf{V}^\alpha |J| d\mathcal{V} \\ &+ \frac{m_0}{M_i} \oint\limits_{\partial\mathcal{V}_i} |J| x^\alpha (v^\beta - \mathsf{V}^\beta) d\sigma_\beta. \end{aligned}$$

Here, v^β are the velocity components at motion of the points of the isosurface $\partial\mathcal{V}_i$ at its own motion and at changes of its geometry. The calculation details are given in Appendix B. Assuming that points of the isosurface $\partial\mathcal{V}_i$ are transported by the field V, we automatically obtain that at this boundary $v^\alpha = \mathsf{V}^\alpha$. As a result, we finally get

$$V_i^\alpha = \frac{dX_i^\alpha}{dt} = \frac{m_0}{M_i}\int\limits_{\mathcal{V}_i} \mathsf{V}^\alpha |J| d\mathcal{V}, \qquad (64)$$

that is, the average motion velocity of the particle as a whole, connected with the transport field V, is equal to the particle volume-averaged value of this field. Similarly, for the particle's average acceleration we find (see Appendix B)

$$A_i^\alpha = \frac{d^2X_i^\alpha}{dt^2} = \frac{m_0}{M_i}\int\limits_{\mathcal{V}_i}\left(\frac{\partial}{\partial t}\mathsf{V}^\alpha + \mathsf{V}^\beta\frac{\partial}{\partial x^\beta}\mathsf{V}^\alpha\right)|J|d\mathcal{V}. \qquad (65)$$

Here, the quantity in parentheses in the integrand is the Euler local acceleration of a fluid medium with the velocity field V. Therefor the vector $\mathbf{A}_i$ with components given by (65) is the particle's average acceleration. Therefore, the relation (65) can now be formally considered as the Newtonian equation of particle dynamics by writing it in the following form:

$$M_i\frac{d^2X_i^\alpha}{dt^2} = \mathsf{F}_i^\alpha, \qquad (66)$$

where $\mathcal{F}_i^\alpha$ are components of the vector F of the summed force applied to the particle,

$$\mathsf{F}_i^\alpha = m_0\int\limits_{\mathcal{V}_i}\left(\frac{\partial}{\partial t}\mathsf{V}^\alpha + \mathsf{V}^\beta\frac{\partial}{\partial x^\beta}\mathsf{V}^\alpha\right)|J|d\mathcal{V}. \qquad (67)$$

It is important that the inertial mass M_i is calculated here as an integral of the mass density of points of the hypersurface $\mathcal{V}^3$ over the particle volume. But it is this mass that appears in the dynamic equations of the fundamental gravitational field (20) and (52). So in this theory there is no need to introduce a special postulate on equivalence of gravitational and inertial forces. The equality of inertial and gravitational masses holds a automatically in this theory if the gravitational force in the averaged equations appears in the form of the averaged free-fall acceleration $\overline{\mathbf{g}}$ in the fundamental gravitational field with strength $\mathbf{g}$ (21).

13. FORCES ACTING ON A PARTICLE

To be able to compare (66) with the equations of particle dynamics in classical physics, it is necessary to show that the force F takes the form of standard forces, such as gravitational and electromagnetic ones. To do that, it is necessary to represent (67) in a form suitable for solving such a problem [1, 3]. Let us formally present the transfer field as a set of two fields:

$$\mathsf{V} = \nabla\chi - \gamma_0\mathsf{A}, \qquad (68)$$

where $\chi(\mathbf{x},t)$ and $\mathsf{A}(\mathbf{x},t)$ are some auxiliary fields, a scalar one, $\chi(\mathbf{x},t)$, and a vector one, $\mathsf{A}(\mathbf{x},t)$. The constant γ_0 has been introduced for the final result to take a form known from electrodynamics. This splitting doe not impose any restrictions on V. Then, using the standard relations from vector analysis (see Appendix B) used in hydrodynamics, we find:

$$\frac{\partial}{\partial t}\mathsf{V}^\alpha + \mathsf{V}^\beta\frac{\partial}{\partial x^\beta}\mathsf{V}^\alpha = -\gamma_0\left(\frac{\partial}{\partial t}\mathsf{A}^\alpha + \frac{\partial\Phi}{\partial x^\alpha}\right) - \gamma_0[\mathrm{curl}\,\mathsf{A}\times\mathsf{V}]^\alpha + \frac{\partial}{\partial x^\alpha}U, \qquad (69)$$

with

$$U = \frac{1}{2}\mathsf{V}^2 + \frac{\partial}{\partial t}\chi + \gamma_0\Phi, \qquad (70)$$

and the scalar field Φ has been introduced for convenience of further interpretation of the relations (69). The meaning of splitting (68) becomes clear if we interpret A as the vector potential of the classical electromagnetic field, which is no longer fundamental. Then the scalar field Φ should be considered as the potential of the electric field, and the fields E^α and H^α, defined as

$$E^\alpha = -\gamma_0\left(\frac{\partial}{\partial t}\mathsf{A}^\alpha + \frac{\partial\Phi}{\partial x^\alpha}\right), \quad H^\alpha = [\mathrm{curl}\,\mathsf{A}]^\alpha, \qquad (71)$$

as the strengths of electric and magnetic fields of classical electrodynamics. With such interpretation, $\chi(\mathbf{x},t)$ should be treated as the action function of classical mechanics, and Eq. (70) written in the form

$$\frac{\partial}{\partial t}\chi + \frac{1}{2}(\nabla\chi - \gamma_0\mathsf{A})^2 + \gamma_0\Phi - U = 0, \qquad (72)$$

as the Jacobi equation of classical mechanics with respect to the action of $\chi(\mathbf{x},t)$ for a particle with unit mass and charge moving in a magnetic field with vector potential A and in an electric field with potential Φ. For the function $U(\mathbf{x},t)$, the only remaining role iis that of the potential of the classical gravitational field.

Now we can get an explicit view of the averaged force acting on a particle. To do so, we additionally introduce the following representation of classical fields:

$$\mathsf{V} = V^\alpha(t) + \mathsf{V}', \quad \mathsf{A} = \overline{\mathsf{A}}(t) + \mathsf{A}',$$
$$\mathbf{H} = \overline{\mathbf{H}}(t) + \mathbf{H}', \quad \mathbf{E} = \overline{\mathbf{E}}(t) + \mathbf{E}',$$
$$U = \overline{U}(t) + U'.$$

Here, $\overline{\mathsf{A}}$, $\overline{\mathbf{E}}$, $\overline{\mathbf{H}}$, and $\overline{U}$ are fields obtained by averaging over $\mathcal{V}_i$ with the density $\rho = \mathcal{M}(\mathbf{e})|J|$, and fields with a prime are field deflections from their average value at each point of space. Substituting the fields in this form into Eqs. (67), we obtain the relation

$$\mathsf{F}_i^\alpha = \overline{E}^\alpha - \gamma_0[\mathbf{H}\times\mathbf{V}]^\alpha + \nabla_X \overline{U} + \mathsf{F}_q^\alpha, \qquad (73)$$

where ∇_X is a gradient by he averaged coordinates X^α, and

$$\mathsf{F}_q^\alpha = -\gamma_0 \int\limits_{\mathcal{V}_i} [\mathbf{H}'\times\mathsf{V}']d\mathcal{V},$$

is a correction to the averaged forces due to a correlation of fields' deflection from the average. In such a notation,

$$M_i \frac{d^2 X_i^\alpha}{dt^2} = \overline{E}^\alpha - \gamma_0[\mathbf{H}\times\mathbf{V}]^\alpha + \nabla_X \overline{U} + \mathsf{F}_q^\alpha, \quad (74)$$

it becomes clear that Newton's equations (74) are the equations of motion of classical mechanics for a charged particle in an electromagnetic field and in a field with the scalar potential $\overline{U}(\mathbf{X}(t), t)$. The corrections F_q to "classical" forces acting on a particle have the same meaning as quantum corrections to the mean values of forces known in quantum theory [29]. It remains to show that the whole ideology with averaged particle motion can be considered as a geometric interpretation of quantum theory.

14. QUANTUM EQUATIONS OF PARTICLE DYNAMICS

Consider the function Ψ of the form

$$\Psi = \sqrt{\mathcal{M}(\mathbf{e})|J|}e^{i\chi/\hbar}. \qquad (75)$$

Here i is the imaginary unit, and $\hbar$ is the Planck constant, introduced formally. By direct calculations with (72) and the conservation law for $\rho = \mathcal{M}(\mathbf{e})|J|$ we verify that this function satisfies the Schrödinger equation

$$i\hbar\frac{\partial\Psi}{\partial t} = \frac{1}{2}\Big(-i\hbar\nabla - \gamma_0\mathsf{A}\Big)^2\Psi + (\gamma_0\Phi - U_G)\Psi = 0 \qquad (76)$$

for a quantum particle moving an a classical electromagnetic field and an additional field with the potential

$$U_G = U - \frac{\hbar^2}{2}\frac{\Delta\sqrt{|J|}}{\sqrt{|J|}},$$

where Δ is the Laplace operator, $\Delta = \sum_{\alpha=1}^3 \dfrac{\partial^2}{\partial x_\alpha^2}$. Similar calculations were first presented in [30], indicating a connection of quantum mechanics with hydrodynamics. Equation (76) practically does not differ from the Schrödinger equation of quantum mechanics, except for a nonobvious interpretation of the potential U_G. This allows us to believe that the present theory explains quantum theory from a geometric point of view.

First of all, Born's statistical postulate (62) receives here a geometric interpretation. The wave function defined using Eq. (75) automatically leads to a geometric interpretation of this postulate:

$$|\Psi|^2 = \mathcal{M}(\mathbf{e})|J|. \qquad (77)$$

On the right, there are quantities characterizing the non-Euclidean nature of $\mathcal{V}^3$ in terms of the properties of marker fields. It seems that this completely excludes any probabilistic treatment of this postulate.

Meanwhile, it is well known that the statistical approach in quantum theory in most problems leads to experimentally correct calculations. Difficulties arise in some special situations, for example, when analyzing Bell's inequalities [10], when one has to assume that the quantum probability theory differs from the classical one. But Eq. (77), in a certain sense, gives an understanding of why the statistical approach is effective in most situations. This understanding is based on the above-mentioned conditional duality of particles as basic topological cells. On the one hand, particles are extended objects, but a central place in their description is occupied by critical points of the fundamental potential $\mathcal{F}$, i.e., point objects, attributed to point charges. In this theory, these special points correspond to well-defined markers. It remains to recall that the function $|J|$ is the density of markers. In a quantum experiment, it is impossible to accurately determine using $|J|$, where this or that critical point $\mathcal{F}$ is located. The function $|J|$ does not contain exact information on the position of these points, but we can assume that a critical point is most likely where the density of markers is higher. This explains why the statistical point of view is successful at calculations in quantum theory, but mainly for simple particles like an electron. It also explains why the statistical approach faces difficulties for more complex particles, such as photons, whose structure contains several critical points.

The second most important postulate in quantum theory is the continuity postulate for the complete function. From the viewpoint of (77), the continuity postulate means continuity of the function $\mathcal{M}(\mathbf{e})|J|$ that means that at the boundaries of the base cells, for short, 1 and 2, which are special isosurfaces of $\mathcal{F}$, it should hold

$$\mathcal{M}(\mathbf{e})|J|\Big|_1 = \mathcal{M}(\mathbf{e})|J|\Big|_2. \qquad (78)$$

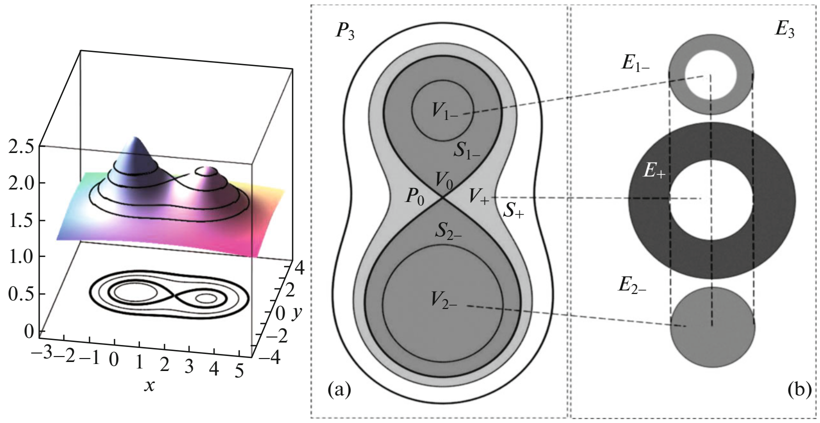


Fig. 10. Changes in the cell image structure (Fig. 2) in marker space when using a single common value of $\mathcal{F}_i$.

However, these relations contradict the boundary conditions for the fundamental field of electrical induction (35). The need to simultaneously fulfill the relations (35) and (78) leads to the requirement that the following conditions must hold at the boundary:

$$\mathcal{M}(\mathbf{e})|\mathbf{e}|\Big|_1 = \mathcal{M}(\mathbf{e})|\mathbf{e}|\Big|_2. \tag{79}$$

These conditions impose restrictions on the way how markers are numbered in each cell. Assuming that the function $\mathcal{M}(\mathbf{e})$ depends only on $R = |\mathbf{e}| = \sqrt{2|\mathcal{F}_i - \mathcal{F}|}$ or is itself continuous, the condition (79) reduces to the continuity of $R = |\mathbf{e}|$. Due to continuity of $\mathcal{F}$, the continuity condition of $|\mathbf{e}|$ is the requirement to choose the same value in all topological cells for $\mathcal{F}_i$:

$$\mathcal{F}_0 = \mathcal{F}_1 = \mathcal{F}_2 = \mathcal{F}_3 = \cdots, \tag{80}$$

where $\mathcal{F}_0$ is chosen arbitrarily. These conditions mean that the image of the maps of almost all simple topological cells to the corresponding sheets $\mathcal{E}_i$ of the marker space will not be balls, but spherical layers with two radii $R_1 < R_2$. For almost all nonempty simple cells, one of the boundary spheres will be the image of the extremum of $\mathcal{F}$ lying in the cell. Figure 10 shows the changes in the structure of cell images shown in Fig. 2 after using the condition (80). As can be seen from the figure, as a result of the "alignment" of values at the extrema of the cells, the radii of their images in $\mathcal{E}^3$ will coincide at the borders, regardless of the sheet $\mathcal{E}_i$ on which these images are located. In this case, there are no changes in the calculations carried out earlier. But at the same time, there happens a general unification of the mass calculation of particles as topological cells.

We will leave beyond the scope of this article the task of explaining the third cornerstone, being the most irrational postulate of quantum theory, the projection postulate [10]. The corresponding explanations require an additional analysis, which lies beyond the conceptual presentation of the proposed theory. Note that the duality of particle structure in this theory also serves as an element of explaining the geometric meaning of this postulate and related problems such as an explanation of interference experiments with electrons and other particles.

Also, a discussion of the operator formalism of quantum theory remains outside the scope of the article. This tool, making it possible to efficiently carry out calculations in quantum theory, is a consequence of a set of postulates, some of which have already been discussed in this and other papers devoted to this theory. Therefore, one can believe that this formalism will be preserved in the new theory, may by with some changes, as an effective calculation tool in quantum dynamics.

15. PROBLEMS OF THE NEW THEORY

In conclusion of this paper, consider some of the most significant problems that do not allow us to treat this theory as an entirely complete physical theory that explains all basic fundamental phenomena from micro- to macrocosm. From a general analysis of the constructions given in this paper, it follows that to explain the properties of matter, the theory introduces a material hypersurface $\mathcal{V}^3$ whose points have the property of massiveness. This attribute of the points of $\mathcal{V}^3$ actually designates its materiality. Tracking the points of $\mathcal{V}^3$ is carried out, as in the theory of continuous media, using marker fields. Using the properties of marker fields, it is possible to describe the properties of the structure of matter, including its electric charge, not as a property of individual points of $\mathcal{V}^3$ but as topological properties of $\mathcal{V}^3$. Similarly, it appears possible to describe the baryonic charge, based on the general Wheeler–Misner idea of a "charge without charge" [21–23]. All charge and gravitational properties of particles are described using fundamental induction and strength fields (**D** and **E**) of both electromagnetic and gravitational fields (**g** and **Z**). Moreover, both these fields are actually a manifestation of the same non-Euclidean property of $\mathcal{V}^3$ and are in fact closely related to each other. The equations of these fields are similar to those equations of classical electrodynamics and gravitation, but describe the properties of $\mathcal{V}^3$ rather than independent fields. This description implements Einstein's idea that fields are properties of space-time itself, but in a way different from GR. In GR, this idea was implemented only for the gravitational field.

The second element of the proposed theory is the use of geometric averaging for an averaged description of particle motion, which coincides in form with the description of quantum mechanics. In the framework of such a description, there emerge averaged equations of motion of particles as extended objects, and the Schrödinger equation. At the same time, the constructed particle dynamics, on the one hand, is based on the initial assumptions of this theory, which a basis of a topological-geometric theory of particle structure, but, on the other hand, it contains another set of electromagnetic fields $\mathbf{E}$, $\mathbf{H}$, etc., different from the basic fields. While the fundamental fields give an ideologically correct idea of particle structure, the second set of fields naturally enters into the Newton and Schrödinger equations. But this second set of fields is connected with the basic fields only indirectly through the marker transport field **V**. It follows from physical considerations that there must be a direct relationship between the fields **D**, **H**, **g**, **Z** and the fields of the second set **A**, $\mathbf{E}$, $\mathbf{H}$, U, etc. The absence of such a connection is an essential problem of this theory.

This problem looks most significant for the gravitational field. In the experiment, the gravitational field manifests itself in as the existence of a free-fall acceleration at every point in space. This means that the field **g** together with the field **Z** should appear in the equations of averaged motion. However, in Eqs. (74) there is only one scalar function U for connection with the gravitational field, and it is not directly related to the fields **g** and **D**.

A way to overcome this difficulty has not yet been found, but a direction in which, apparently, it is possible to find a solution to the problem, was briefly described in [6]. The meaning of the approach, which will presumably make it possible to close the theory in this part, can be described as follows. As mentioned at the beginning of this paper, the hyperplane $\mathcal{P}^3 \in \mathcal{W}^4$ is a mathematical implementation of a reference frame. Such a frame can be distinguished by averaging $\mathcal{V}^3$ with the same geometric density as the

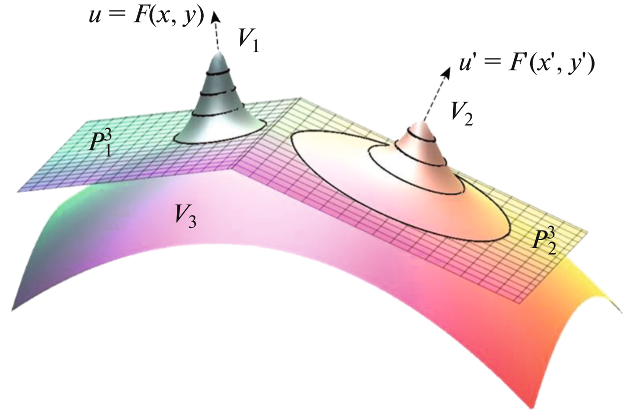


Fig. 11. "Proper" hyperplanes $\mathcal{P}_1^3$ and $\mathcal{P}_2^3$ for two topological cells.

Newtonian equations. Indeed, averaging Eq. (1) with the density $\mathcal{M}(\mathbf{e})|J|$ and using (29), we find:

$$wM_i = \mathcal{F}_i M_i + m_0 \frac{\varepsilon}{2} \int\limits_{V_i} |\mathbf{e}|^2 \mathcal{M}(\mathbf{e})|J| dV.$$

From here we find that the equation of the averaged hypersurface has the form

$$w = \mathcal{F}_i + \frac{m_0}{M_i}\overline{R}. \tag{81}$$

In this theory, any integral of the form

$$I = \int\limits_{V_i} I(\mathbf{e})\mathcal{M}(\mathbf{e})|J| dV$$

is a conserved quantity, $dI/dt = 0$. This follows from the fact that $\mathbf{e}$, by definition, obeys the marker transport equations. Hence, the averaged hyperplane in $\mathcal{W}^4$ is specified by Eq. (81): $w = \text{const}$. Thus, the hyperplane $\mathcal{P}^3$ is distinguished for each individual particle, which belongs to common numbering of markers in a given region of space. However, in reality, matching the marker numbering in different parts of space is a definite problem. Figure 11 presents a two-dimensional illustration of how, in reality, different marker numberings can determine different hyperplanes $\mathcal{P}^3$ that are not parallel to each other.

A general approach to explaining the emergence of the fundamental forces associated with **D**, **g**, etc., in the equations of motion reduces, apparently, to taking into account, both in the averaged Newton equations and the Schrödinger equation, the changes in the "proper" hyperplane $\mathcal{P}^3$ of a particle due to its motion. The result of such a modification should be generalizations of the dynamic equations of a self-gravitating medium (2)–(4), from which we started the presentation of the basic concepts of this theory. However, the description and analysis of this approach go beyond the scope of this article.

One more problem of the proposed theory, to be discussed here, is the problem of geometrodynamics. The meaning of this problem, which was discussed previously in [5–7], is that most of the relations that form the basis of the proposed theory are mathematical identities connecting various properties of marker fields and transfer fields. This means that almost all equations of the theory are determined by the properties of a single function $\mathcal{F}(\mathbf{x}, t)$, for which, however, there is no separate dynamic equation in the theory. This means that all constructions are valid for any smooth function $\mathcal{F}$. But in nature, only one possible form of $\mathcal{F}$ is realized. It follows that for the fundamental potential in theory there must be a separate equation reflecting the physical essence of a specific implementation of the hypersurface $\mathcal{V}^3$ in $\mathcal{W}^4$. This physical essence of $\mathcal{V}^3$, of which we are a part, is not yet available for research in an experiment. Therefore, one can only put forward various hypotheses on the nature of $\mathcal{V}^3$.

For example, the hypersurface can be an analogue of the boundary of two media filling $\mathcal{W}^4$ or some part of it. Another hypothesis may be the assumption that $\mathcal{V}^3$ is a three-dimensional "membrane," itself consisting of particles of some supermatter. A common feature of these hypotheses is that the description of $\mathcal{V}^3$ must be constructed using markers numbering the points of this hypersurface. In this case, we can suppose that for the dynamics of $\mathcal{V}^3$, as an element of $\mathcal{W}^4$, it is possible to write down more general marker transport equations not along $\mathcal{P}^3$, but in the entire space. The general idea of this approach was outlined in [5–7] and in the recent paper [9].

The general result of the constructions carried out in these papers is that the dynamics equation for $\mathcal{F}$ can be reduced, under sufficiently general considerations, to the form

$$\nabla\mathcal{F} - \frac{\partial}{\partial t}\left(\frac{1}{c^2(\mathcal{F})}\frac{\partial}{\partial t}\right)\mathcal{F} = P(\mathcal{F}), \qquad (82)$$

in which the functions $c(\mathcal{F})$ and $P(\mathcal{F})$ require an experimental justification. The form of equation itself coincides with that of the generalized equation for vibrations of an infinitely thin 3D elastic inhomogeneous membrane in 4D space. In this case, the function $c(\mathcal{F})$ is an analogue of the local velocity of elastic waves of the membrane, and $P(\mathcal{F})$ is the normal pressure on the membrane from external forces. This description scheme fits both the hypothesis of the membrane itself and the hypothesis of a boundary of two media. Perhaps more complete information on the nature of the dynamics of $\mathcal{V}^3$ will be brought by a solution of the first of the problems outlined here.

Appendix A

From the general definition of an inverse matrix one can obtain the following set of identities that can be used while deriving the induction equation for the field K:

$$\frac{\partial x^\alpha}{\partial e^a}\frac{\partial e^a}{\partial x^\beta} = \delta^\alpha_\beta,$$

$$\frac{\partial x^\alpha}{\partial e^a}\frac{\partial e^b}{\partial x^\alpha} = \delta^b_a,$$

$$\frac{\partial x^\alpha}{\partial e^a} = \frac{1}{|J|}\varepsilon_{abc}\varepsilon^{\alpha\beta\gamma}\frac{\partial e^b}{\partial x^\beta}\frac{\partial e^c}{\partial x^\gamma} = \frac{1}{|J|}\varepsilon_{abc}[\nabla e^b \times \nabla e^c]^\alpha. \qquad (A.1)$$

Using the fact that J is the determinant of the Jacobi matrix (8), we arrive at one more identity:

$$\varepsilon_{abc}[\nabla e^b \times \nabla e^c]\frac{\partial e^a}{\partial x^\beta} = J\frac{\partial x^\alpha}{\partial e^a}\frac{\partial e^a}{\partial x^\beta} = J\delta^\alpha_\beta. \qquad (A.2)$$

From the well-known properties of the Levi-Civita antisymmetric symbol we have the following identity for an arbitrary antisymmetric matrix with the elements $\lambda_{\alpha\beta}$:

$$\varepsilon^{\alpha\beta\gamma}\varepsilon_{\gamma\mu\nu}\lambda_{\alpha\beta} = \det\begin{vmatrix}\delta^\alpha_\gamma & \delta^\beta_\gamma & \delta^\gamma_\gamma \\ \delta^\alpha_\mu & \delta^\beta_\mu & \delta^\gamma_\mu \\ \delta^\alpha_\nu & \delta^\beta_\nu & \delta^\gamma_\nu\end{vmatrix}\lambda_{\alpha\beta} = \lambda_{\alpha\beta}\left(\delta^\alpha_\mu\delta^\beta_\nu - \delta^\alpha_\nu\delta^\beta_\mu\right) = 2\lambda_{\mu\nu}. \qquad (A.3)$$

A result of the last identity is the relation

$$\varepsilon^{\alpha\beta\gamma}\lambda_{\alpha\beta} = \varepsilon^{\alpha\beta\gamma}\varepsilon_{abc}e^a\frac{\partial e^b}{\partial x^\alpha}\frac{\partial e^c}{\partial x^\beta} = \mathsf{K}^\gamma. \qquad (A.4)$$

Appendix B

Using (7), we can write:

$$\int\limits_{\mathcal{V}_i} x^\alpha\frac{\partial}{\partial t}\Big(\mathcal{M}(\mathbf{e})|J|\Big)d\mathcal{V} = -\int\limits_{\mathcal{V}_i} x^\alpha\frac{\partial}{\partial x^\beta}\Big(\mathsf{V}^\beta\mathcal{M}(\mathbf{e})|J|\Big)d\mathcal{V}.$$

To calculate the r.h.s. of the latter relation, we will use the Ostrogradsky-Gauss theorem. As a result, we find

$$\int\limits_{\mathcal{V}_i} x^\alpha\frac{\partial}{\partial x^\beta}\Big(\mathsf{V}^\beta\mathcal{M}(\mathbf{e})|J|\Big)d\mathcal{V}$$

$$= -\int\limits_{\mathcal{V}_i} \mathsf{V}^\alpha \mathcal{M}(\mathbf{e})|J|d\mathcal{V} + \oint_{\partial\mathcal{V}_i} \mathcal{M}(\mathbf{e})|J|x^\alpha \mathsf{V}^\beta d\sigma_\beta$$

$$= -\int\limits_{\mathcal{V}_i} \mathsf{V}^\alpha \mathcal{M}(\mathbf{e})|J|d\mathcal{V} + \oint_{\partial\mathcal{V}_i} \mathcal{M}(\mathbf{e})|J|x^\alpha \mathsf{V}^\beta d\sigma_\beta.$$

For the second time derivative of the average coordinate we have:

$$\frac{d^2 X_i^\alpha}{dt^2} = \int\limits_{\mathcal{V}_i} \left(\frac{\partial V^\alpha}{\partial t} \mathcal{M}(\mathbf{e})|J| + V^\alpha \frac{\partial}{\partial t}\Big(\mathcal{M}(\mathbf{e})|J|\Big) \right) d\mathcal{V} + \oint_{\partial\mathcal{V}_i} \mathcal{M}(\mathbf{e})|J| V^\alpha v^\beta d\sigma_\beta,$$

where v^β are velocity components of moving boundary points. By analogy, we have

$$\int\limits_{\mathcal{V}_i} V^\alpha \frac{\partial}{\partial t}\Big(\mathcal{M}(\mathbf{e})|J|\Big) d\mathcal{V} = -\int\limits_{\mathcal{V}_i} \frac{\partial V^\alpha}{\partial x^\beta} \mathsf{V}^\beta \mathcal{M}(\mathbf{e})|J|d\mathcal{V} + \oint_{\partial\mathcal{V}_i} \mathcal{M}(\mathbf{e})|J| \mathsf{V}^\alpha \mathsf{V}^\beta d\sigma_\beta.$$

Substituting the latter relation to the expression for the second derivative of X_i^α, we finally find

$$\frac{d^2 X_i^\alpha}{dt^2} = \int\limits_{\mathcal{V}_i} \left(\frac{\partial}{\partial t} \mathsf{V}^\alpha + \mathsf{V}^\beta \frac{\partial}{\partial x^\beta} \mathsf{V}^\alpha \right) |J| d\mathcal{V} + \oint_{\partial\mathcal{V}_i} \mathcal{M}(\mathbf{e})|J| \mathsf{V}^\alpha \Big(v^\beta - \mathsf{V}^\beta \Big) d\sigma_\beta.$$

The following identity is often used in hydrodynamics [31]:

$$\frac{\partial}{\partial t} \mathsf{V}^\alpha + \mathsf{V}^\beta \frac{\partial}{\partial x^\beta} \mathsf{V}^\alpha = \frac{\partial}{\partial t} \mathsf{V}^\alpha + [\mathrm{curl}\, \mathbf{V} \times \mathbf{V}]^\alpha + \frac{\partial}{\partial x^\alpha} |\mathbf{V}|^2.$$

Substituting Eq. (68) to this identity, we arrive at Eq. (69).

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CONFLICT OF INTEREST

The author declares that he has no conflicts of interest.

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