ISSN 0202-2893, Gravitation and Cosmology, 2020, Vol. 26, No. 2, pp. 153–161. © Pleiades Publishing, Ltd., 2020.

Cosmological Solutions in the Tensor-Multi-Scalar Theory of Gravity with the Higgs Potential

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Received February 9, 2020; revised February 19, 2020; accepted February 26, 2020

Abstract—We consider a generalization of Higgs inflation, based on cosmology of general relativity (GR), to the case of a two-field model of the tensor-multiscalar theory of gravity (TMS TG). Cosmological solutions are found in the case where a scalar field with the Higgs potential, as a source of TMS TG is in the slow-rolling mode. Solutions with power and exponential-power-law evolution of the scale factor are obtained for various limiting forms of the Higgs potential.

DOI: 10.1134/S0202289320020048

1. INTRODUCTION

An important role in the development of modern cosmology belongs to the inflationary stage in the early epoch of the Universe evolution, which has become a necessary feature of the Big Bang model, which leads to a solution of many problems of the Friedmannian standard cosmology, including the problems of the horizon, flatness and large scale structure formation. The models of cosmological inflation are usually based on the consideration of Einstein's gravity, minimally coupled to a self-interacting scalar field [1]. The field characteristics, including its mass and the shape of its potential, determine the type of the phenomenological model that must necessarily lead to agreement with the observational data. However, quite often the questions of the shape of the potential and the physical essence of the scalar field itself do not receive a sufficient justification from the standpoint of fundamental physics. An important exception is the nonminimal Higgs inflation proposed by Bezrukov and Shaposhnikov [2] in 2008. For many researchers, this work has served as a starting point for further extensions of the model, such as Higgs inflation with a dilaton [3, 4], hybrid Higgs inflation [5], brane inflation with a Higgs potential [6], Higgs inflation with a scalar [7]. It should be noted that the concept of a "scalaron" was introduced in the Starobinsky model [8] (reliably consistent with the observational data) in his scenario in the framework of modified $R + R^2$ gravity. Thus we notice a tendency to introduce additional scalar fields into consideration, which is connected with the necessity to eliminate the contradiction with experiments on the electroweak interactions [9, 10].

The inclusion of additional fields into consideration leads to the idea to investigate the multi-field theory of gravity [11] (or the chiral cosmological model [12]), which is similar to the tensor-multiscalar theory of gravity. One should note the possible transition between the scalar tensor theory of gravity and other modified theories, including $f(R)$ gravity [13], multidimensional gravity [14], gravity with a kinetic curvature scalar [15, 16].

In this paper, we consider an inflationary scenario with a Higgs potential in the tensor-multiscalar theory of gravity (TMS TG), which is a natural extension of scalar-tensor gravity. To find solutions in such a model, we use the Anzatz method [17], which has proven itself well in solving similar problems. Solutions for the Higgs field are searched for in the range between two limiting cases, with the scalar field being in the slow-rolling mode.

This study is a continuation of [18], which considered two partitions (Anzätze) for the exponential and power-law scale factors and various choices of the scalar field potential.

The content of the article is distributed as follows. The second section discusses a transition from the Standard model of particle physics nonminimally

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coupled to gravity to the Einstein gravity with a scalar field. During the transition to the Einstein frame, the GR sector is generalized to TMS TG, which is justified by the assumption that there are additional fields such as the scalaron, the inflaton, and the dilaton in the original Standard model. Further, we notice the freedom to choose one more conformal transformation introduced by TMS TG. Next, we point out the choice of the space-time metric for a homogeneous and isotropic Universe and the target space metric corresponding to the introduction of two scalar fields, and we write down the equations of cosmological dynamics in this case.

The third section presents the limiting values of the Higgs potential used in the study of inflationary models.

The fourth section provides a description of the method and presents solutions (in the slow-rolling mode) for power-law inflation in the limit of small values of the Higgs field and deviations of the conformal transformation function from unity. Another limiting case when using the superpotential method with respect to the Higgs potential leads to an exponential-power-law evolution of the scale factor. To compare the properties of the gravitational scalar fields with models of particle physics, Section 5 presents the masses determined from the found potentials in a standard way.

2. THE MODEL AND THE CHOICE OF MATERIAL ACTION

Bezrukov and Shaposhnikov [2] considered the scalar sector of the Standard Model of particle physics interacting with gravity in a nonminimal manner. Choosing the unitary gauge and neglecting all gauge interactions, the action is presented in the J-frame (the Jordan conformal frame) as

$$S_J = \int d^4x\sqrt{-\tilde{g}}\Big(- \frac{M^2+\xi h^2}{2}\tilde{R} + \frac{1}{2}\tilde{g}^{\mu\nu}\partial_\mu h \partial_\nu h - \frac{\lambda}{4}(h^2-v^2)^2\Big), \qquad (1)$$

where $M \simeq M_P$ with high accuracy, the field h is the Higgs field in the unitary gauge, $H = h/\sqrt{2}$, $\tilde{()}$ denotes consideration in the J-frame, $v = 246$ GeV $= 1.1\times 10^{-16} m_p$ is the vacuum mean value of the Higgs field, and the factor λ is equal to 0.1 [19].

A transition to the E-frame (the Einstein conformal frame) is carried out using the conformal transformation (see, e.g., [28])

$$g_{\mu\nu} = \omega^2 \tilde{g}_{\mu\nu}, \qquad (2)$$

$$\omega^2 = 1 + \frac{\xi h^2}{M_P^2} \qquad (3)$$

The conformal transformation (2), (3) leads to a redefinition of the original scalar field h according to the relation

$$\frac{d\chi}{dh} = \sqrt{\frac{\omega^2 + 6\xi^2 h^2\, M_P^2}{\omega^4}}. \qquad (4)$$

As a result, the new field χ describes the Higgs field in the E-frame. Thus we have made a transition from the action (1) to the E-frame action

$$S_E = \int d^4x\sqrt{-g}\Big(- \frac{M^2}{2}R + \frac{\partial_\mu\chi\partial^\mu\chi}{2} - U_H(\chi)\Big), \qquad (5)$$

where the Higgs potential U_H is defined as the field function χ taking into account the relation (4) and has the form

$$U_H(\chi) = \frac{1}{\omega(\chi)^4}\frac{\lambda}{4}(h(\chi)^2 - v^2)^2. \qquad (6)$$

There are at least two ways to introduce additional fields into the theory. The first way is to introduce fields as sources of the gravitational field, as is done for the Higgs model with the dilaton [3]or the scalar [7]. Another option is to switch to the scalar-tensor theory of gravity or, with the inclusion of a few scalar fields, to the tensor-multiscalar theory of gravity [20]. In this case, it is suggested to consider the additional fields of the dilaton and/or scalaron type as gravitational scalar fields. Let us note that this transition is easily accomplished if we work in the natural system of units, where $8\pi G = \varkappa = M_P^{-2} = 1$.

Following the approach proposed in Damour and Esposito-Farese (1992) [20], we consider the TMS TG in the E-frame without a nonminimal interaction of the scalar curvature with the gravitational scalar fields, when the action of the matter field as a source of gravity is considered in the "physical" metric $g^\star_{\mu\nu}$, conformally related to the metric $g_{\mu\nu}$ in the E-frame by the Weyl transformation $g^\star_{\mu\nu} = \Omega^2(\varphi) g_{\mu\nu}$. Continuing the studies presented in [18], we consider a tensor-multi-scalar model of the theory of gravity with the action

$$S = \int d^4x\sqrt{-g}\left[\frac{R}{2} - \frac{1}{2}g^{\mu\nu}h_{AB}\varphi^A_{,\mu}\varphi^B_{,\nu} - W(\varphi^C)\right] + S_m[\chi_m, \Omega^2(\varphi^C)g_{\mu\nu}]. \qquad (7)$$

Our notations here are mostly the same as those used in [18]: $\varkappa = 1$ is the Einstein gravitational constant, R is the scalar curvature, $g = \det(g_{\mu\nu})$. To shorten the record, we use $\varphi_{,\mu} = \partial_\mu\varphi$. Greek indexes $\mu, \nu, ... = 0, 1, 2, 3$ correspond to the space-time coordinates. Capital Latin indexes $A, B, C, ... = 1, 2...N$ specify N scalar fields. Furthermore, the set of scalar fields $\{\varphi^1, \varphi^2, ...\varphi^N\}$ will be denoted as $\varphi := \{\varphi^1, \varphi^2 ...\varphi^N\}$. We use the Planck mass M_P^2 instead of the Einstein gravitational constant $\varkappa$, with $M_P^2 = \varkappa^{-1}$.

The gravitational part of the action (7) in the absence of S_m corresponds to the chiral cosmological model (CCM) while selecting natural units, including $\varkappa = M_P^{-2} = 1$. Thus the solutions obtained in a number of works for CCM [21–23] can be considered as vacuum solutions in TMS TG.

Let us assume that the material (non-gravitational) part of S_m is described by the scalar field χ with the Higgs potential U_H (6) in the E-frame. In this case, one more conformal transition $g^{\star}_{\mu\nu} = \Omega^2(\varphi) g_{\mu\nu}$ must be performed to add the field to the action (7). Then the model (7) with the Higgs field included in the action takes the form

$$S = \int d^4x\sqrt{-g}\left[\frac{R}{2} - \frac{1}{2}g^{\mu\nu}h_{AB}\varphi^A_{,\mu}\varphi^B_{,\nu} - W(\varphi)\right] + \int d^4x\sqrt{-g}\Omega^4(\varphi) \times \left[-\frac{1}{2}g^{\mu\nu}_{\star}\chi^{\star}_{,\mu}\chi^{\star}_{,\nu} - U_{\star}(\chi^{\star})\right]. \quad (8)$$

The transition to the new canonical field and its potential can be carried out with the transformations

$$U_H(\chi) = \Omega^4(\varphi)U_{\star}(\chi^{\star}) \quad (9)$$

$$\frac{d\chi}{d\chi_{\star}} = \Omega(\varphi) \quad (10)$$

$$g^{\mu\nu}_{\star} = \Omega^{-2}(\varphi)g^{\mu\nu}. \quad (11)$$

The choice of the Higgs potential $U_H(\chi)$ (9) is made due to the fact that its transformed form (6) corresponds to the Einstein picture, as well as the gravitational component in (8) (the first integral term).

Thus, taking into account the specified substitutions (9)–(11), the action (8) takes the form

$$S = \int d^4x\sqrt{-g}\Bigg[\frac{R}{2} - \frac{1}{2}g^{\mu\nu}h_{AB}\varphi^A_{,\mu}\varphi^B_{,\nu} - W(\varphi) - M_P^{-2}\left(\frac{1}{2}g^{\mu\nu}\chi_{,\mu}\chi_{,\nu} - U_H(\chi)\right)\Bigg]. \quad (12)$$

In what follows $M_P^2 = 1$. The model (12) is a TMS TG with a source in the form of a self-interacting scalar field with the original Higgs potential. Assuming that the source of gravity is specified in the same space-time as in the case of (5), we can suppose that $\Omega^2(\varphi) \simeq 1$. However, we preserve the possibility that Ω changes in time, remaining close to unity [2].

The scalar component of the action (7) of the gravitational field is selected in the representation of a two-component CCM with the the target space metric

$$d\sigma^2 = h_{11}d\phi^2 + h_{22}(\phi,\psi)d\psi^2. \quad (13)$$

Here, for the chiral fields, we are using the notations $\varphi^1 = \phi$, $\varphi^2 = \psi$ and choose the Gaussian coordinate system, $h_{11} = \pm 1$.

We write the metric of the homogeneous isotropic universe in the Friedmann–Robertson–Walker (FRW) form

$$ds^2 = -dt^2 + a^2(t)\left(\frac{dr^2}{1-\epsilon r^2} + r^2d\theta^2 + r^2\sin^2\theta d\varphi^2\right), \quad (14)$$

where $\epsilon = -1, +1, 0$, which corresponds to an open, closed or spatially flat universe, respectively. Let us note that the case of nonzero ϵ may be treated as a spatially flat FRW model with a perfect fluid having the equation of state $p = -3\rho$, $\rho = -\epsilon/(3a^2)$ [24].

Varying the action (8) with respect to the metric and the fields, we obtain a set of equations in the class of metrics (13), (14) having the following form [18]:

$$3H\dot{\psi}h_{22} + \partial_t(h_{22}\dot{\psi}) - \frac{1}{2}\frac{\partial h_{22}}{\partial\psi}\dot{\psi}^2 + \frac{\partial W(\phi,\psi)}{\partial\psi} = \frac{\partial \ln\Omega(\phi,\psi)}{\partial\psi}(\dot{\chi}_{\star}^2 + 4U_{\star}(\chi)), \quad (15)$$

$$\ddot{\phi}h_{11} + 3H\dot{\phi}h_{11} - \frac{1}{2}\frac{\partial h_{22}}{\partial\phi}\dot{\psi}^2 + \frac{\partial W(\phi,\psi)}{\partial\phi} = \frac{\partial \ln\Omega(\phi,\psi)}{\partial\phi}(\dot{\chi}_{\star}^2 + 4U_{\star}(\chi)), \quad (16)$$

$$H^2 = \frac{1}{3}\left[\frac{1}{2}h_{11}\dot{\phi}^2 + \frac{1}{2}h_{22}\dot{\psi}^2 + W(\phi,\psi)\right] + \frac{1}{3}\left(\frac{1}{2}\dot{\chi}_{\star}^2 + U_{\star}(\chi)\right) - \frac{\epsilon}{a^2}, \quad (17)$$

$$\dot{H} = -\left[\frac{1}{2}h_{11}\dot{\phi}^2 + \frac{1}{2}h_{22}\dot{\psi}^2\right] - \dot{\chi}_{\star}^2 + \frac{\epsilon}{a^2}, \quad (18)$$

$$\ddot{\chi}^{\star} + 3H^{\star}\dot{\chi}^{\star} + U^{\star}(\chi)_{,\chi} = 0. \quad (19)$$

The set of equations (15)–(19) describes the cosmological dynamics of the model under consideration. Let us note that at scaling, such that $\Omega(\phi,\psi) = \mathrm{const}$, the Higgs potential affects the dynamics of the chiral fields ϕ and ψ only through the Hubble parameter H.

The consequences of Eqs. (17), (18) may be divided into the equations for the kinetic and potential parts:

$$K(t) = \frac{1}{2}h_{11}\dot{\phi}^2 + \frac{1}{2}h_{22}(\phi,\psi)\dot{\psi}^2 + \dot{\chi}_\star^2 = \frac{\epsilon}{a^2} - \dot{H}, \quad (20)$$

$$W(t) = \left[\dot{H} + 3H^2 + 2\frac{\epsilon}{a^2} - U_\star(\chi)\right]. \quad (21)$$

These equations will be useful in the construction of decompositions (Ansätze).

3. LIMITING VALUES OF THE HIGGS POTENTIAL

The conformal transformation (2)–(4) from the acceptable model (from the point of view of scalar field renormalization in curved space) with the Higgs potential (6), implemented for the first time in [2], turned out to be rather complicated for solving the equations of cosmological dynamics. To simplify the situation, we will use some approximations that have been considered both in the pioneering article [2]and in the subsequent papers (see, e.g., [19]).

The dependence (6) of the potential $U_H(\chi)$ on the scalar field χ in the E-frame is specified as follows:

$$U_H(\chi) = \frac{1}{\omega^4}\frac{\lambda}{4}(h(\chi)^2 - v^2)^2.$$

Let us consider three approximations proposed in [31] and [19]. It is in these ranges that the Higgs inflation with the potential (6) is considered, and we will use these limits in order to find solutions to the cosmological dynamics equation in TMS TG:

(1) For small values of the field $h \ll \sqrt{2/3}M_p/\xi$ and $\omega^2 \simeq 1$, from which it follows

$$\chi \simeq \pm h, \quad |\chi| \ll \sqrt{\frac{2}{3}}\frac{M_P}{\xi}, \quad (22)$$

$$U_H(\chi) \simeq \frac{\lambda}{4}\chi^4. \quad (23)$$

(2) Under the condition

$$\sqrt{\frac{2}{3}}\frac{M_P}{\xi} \ll h \ll \frac{M_P}{\sqrt{\xi}},$$

we obtain

$$\chi \simeq \pm\sqrt{\frac{2}{3}}\frac{\xi h^2}{M_P},$$

$$\sqrt{\frac{2}{3}}\frac{M_P}{\xi} \ll |\chi| \ll \sqrt{\frac{3}{2}}M_P \quad (24)$$

$$U_H(\chi) \simeq \frac{\lambda M_P^2}{6\xi^2}\chi^2, \quad (25)$$

(3) At the values $h \gg M_P/\sqrt{\xi}$ we obtain

$$h \simeq \frac{M_P}{\sqrt{\xi}}\exp\left(\frac{\chi}{\sqrt{6}M_P}\right), \quad (26)$$

or

$$\chi \simeq \pm\sqrt{6}M_P\log\left(\frac{\sqrt{\xi}h}{M_P}\right), \quad (27)$$

$$U_H(\chi) = \frac{\lambda M_P^4}{4\xi^2}\left[1 + \exp\left(\frac{-2\chi}{\sqrt{6}M_P}\right)\right]^{-2}. \quad (28)$$

A good analytic approximation to the potential, which can simultaneously describe the approximations (25) and (28), has the form

$$U_H(\chi) = V_0\left[1 - \exp\left(\frac{-2\chi}{\sqrt{6}M_P}\right)\right]^2, \quad (29)$$

where the value of V_0 is such that [19]:

$$V_0 = \frac{\lambda M_P^4}{4\xi^2} = 9.6\times 10^{-11}M_P^4. \quad (30)$$

For the analysis of theoretical predictions and their confrontation to the observational data, we invoke the slow-rolling mode for the Higgs field. In this case, taking into account the requirements $|\dot{\chi}^2| \ll V(\chi)$ and $|\ddot{\chi}| \ll H|\dot{\chi}|$, Eq. (19) takes the form

$$3H\dot{\chi} + U(\chi)_{,\chi} = 0. \quad (31)$$

At that, in (17) and (18), the squared time derivative of the field χ, that is, $\dot{\chi}^2$, is elemiinated.

In the framework of our model described by the set of equations (15)–(19), we will consider the potentials (23) and (29), but in order to include them into the TMS TG, it is necessary to carry out one more conformal transition (9).

4. APPROXIMATIONS OF THE MODEL, A CHOICE OF THE ANSATZ AND THE CONFORMAL FACTOR

The decomposition (Ansatz) method for finding the solutions has been described in the monograph [17]. In the framework of this approach, we use the following decomposition of the kinetic and potential components of the equations. They follow from Eqs. (20) and (21). At the same time, for the Higgs field, the slow-rolling condition holds.

Ansatz 1

$$h_{22}(\phi,\psi) = h_{22}(\psi), \quad (32)$$

$$h_{22}(\psi)\dot{\psi}^2 = 2\frac{\epsilon}{a^2} \quad (33)$$

Ansatz 2

$$h_{22}(\phi,\psi) = h_{22}(\phi), \quad (34)$$

$$h_{22}(\phi)\dot{\psi}^2 = 2\frac{\epsilon}{a^2}. \qquad (35)$$

The decomposition for the potential and the chiral component is the same for both cases:

$$h_{11} = \text{const}, \quad h_{11}\dot{\phi}^2 = -2\dot{H}, \qquad (36)$$

$$W(\phi,\psi) = W_1(\phi) + W_2(\phi) + W_3(\psi), \qquad (37)$$

$$\psi(t) = \sqrt{2}t. \qquad (38)$$

The specific features of solutions of the TMS TG set of equations (15)–(19) for each Ansatz, under a specified evolution of the Universe and the potential, are presented in [18].

In our model (8), considered in the metrics (13), (14), a nonminimal interaction is absent. Therefore there is an additional freedom in the choice of the conformal factor $\Omega(\phi,\psi)$. Let $\Omega(\phi,\psi)$ be given in the form

$$\Omega(\phi,\psi) = \exp(A\phi + B\psi), \qquad (39)$$

where $A, B \geq 1$. This choice of the conformal factor allows for simplifying the calculations.

4.1. Small Values of the Field $h \simeq \chi$ and $\Omega^2 \simeq 1$

4.1.1. Power-law evolution of the scale factor, $a(t) = ct^m$. The solution for the Higgs scalar field $\chi(t)$ is found from Eq. (31) for the chosen value of the potential (23) and the scale factor $a(t) = ct^m$:

$$\chi(t) = \frac{1}{t}\sqrt{\frac{3m}{\lambda}}. \qquad (40)$$

The solution for $W_1(\phi)$, $\phi(t)$ does not depend on the choice of the decomposition and is the same for both cases of the decomposition, **Ansatz 1** and **Ansatz 2**. Moreover, they are analogous to the solutions obtained previously in [18]:

$$W_1(\phi) = m(3m-1)\exp\left(-\phi\sqrt{\frac{2}{m}}\right), \qquad (41)$$

$$\phi(t) = \sqrt{2m}\ln t. \qquad (42)$$

Assuming for both **Ansatz 1** and **Ansatz 2**

$$\psi(t) = \sqrt{2}t, \qquad (43)$$

we can obtain formulas for finding solutions for the potentials $W_2(\phi)$, $W_3(\psi)$ and the chiral metric component h_{22}. Under the conditions of **Ansatz 1**, the solution formulas are

$$\dot{W}_2(t) = 4U(\chi)\frac{\partial \ln\Omega}{\partial\phi}\dot{\phi}, \qquad (44)$$

$$\dot{W}_3(t) = 4U^\star(\chi)\frac{\partial \ln\Omega}{\partial\psi}\dot{\psi} - 4H\frac{\epsilon}{a^2}. \qquad (45)$$

For **Ansatz 2**, the formulas are

$$\dot{W}_2(t) = 4U^\star(\chi)\frac{\partial \ln\Omega}{\partial\phi}\dot{\phi} + 2\frac{\epsilon\dot{a}}{a^3}, \qquad (46)$$

$$\dot{W}_3(t) = 4U^\star(\chi)\frac{\partial \ln\Omega}{\partial\psi}\dot{\psi} - 2H\frac{\epsilon}{a^2}. \qquad (47)$$

Substituting the expressions (23) for the Higgs potentials to the resulting equations for the potentials, we obtain the expressions presented in Table 1.

4.2. The Limits $h \gg M_{pl}/\sqrt{\xi}$ and $\sqrt{2/3}M_p/\xi \ll h \ll M_p/\sqrt{\xi}$

As has been already said before, a generalization of the two limits

$$\sqrt{\frac{2}{3}}\frac{M_p}{\xi} \ll h \ll \frac{M_p}{\sqrt{\xi}} \quad \text{and} \quad h \gg \frac{M_{pl}}{\sqrt{\xi}}$$

leads to the exponentially flat potential (29):

$$U(\chi) = V_0\left[1 - \exp(\mu\chi)\right]^2, \qquad (48)$$

where

$$\mu = \frac{-2}{\sqrt{6}M_{pl}}, \quad V_0 = \frac{\lambda M_{pl}^4}{4\xi^2} = 9.6\times10^{-11}{M_p}^4.$$

4.2.1. The Higgs potential as a superpotential. Consider a slightly different approach for finding solutions to the model under study. In this case, we will not specify the Hubble parameter, but consider the exponentially flat potential (48) as a superpotential for determining the evolution of the Universe (the scale factor). Following the methodology presented in [25], let us use the potential (48) as a superpotential for finding the χfield. We write down the Higgs field dynamic equations in the slow-rolling approximation, and, in terms of the superpotential,

$$3H^2 \simeq U(\chi), \quad 3H^2 = W_{SP}, \qquad (49)$$

$$3H\dot{\chi} \simeq -U(\chi)_{,\chi}, \quad 3H\dot{\chi} = -W_{SP}(\chi)_{,\chi}. \qquad (50)$$

Let us note that the slow-rolling approximation acquires an exact form in the superpotential representation when the physical potential is replaced by the superpotential [26, 27]. To determine the field $\chi^\star(t)$ (further on we omit the asterisk), we use the expression

$$\dot{\chi} = -\frac{U(\chi)_{,\chi}}{\sqrt{3U(\chi)}}. \qquad (51)$$

Integrating, we find the time dependence of the field:

$$\chi(t) = -\frac{1}{\mu}\ln\left(-\frac{2\mu^2V_0}{\sqrt{3V_0}}t\right). \qquad (52)$$

Table 1

Partition	Solution
Ansatz 1	$h_{22}(\psi) = \frac{\epsilon 2^m}{c^2\psi^{2m}}, \quad \psi(t) = \sqrt{2}t$ $W_2(\phi) = -\frac{9A\sqrt{m^5}}{2\sqrt{2}\lambda}\exp\left(-\frac{4\phi}{\sqrt{2m}}\right)$ $W_3(\psi) = -\frac{12m^2B}{\lambda}\psi^{-3} - \frac{2\epsilon 2^m}{c^2}\psi^{-2m}$
Ansatz 2	$h_{22}(\phi) = \frac{2\epsilon}{c^2}\exp(-2m\phi), \quad \psi(t) = \sqrt{2}t$ $W_2(\phi) = -\frac{9A\sqrt{m^5}}{2\sqrt{2}\lambda}\exp\left(-\frac{4\phi}{\sqrt{2m}}\right) - \epsilon\exp(-\phi\sqrt{2m})$ $W_3(\psi) - \frac{12m^2B}{\lambda}\psi^{-3} + \frac{\epsilon 2^m}{c^2}\psi^{-2m}$

The Hubble parameter is determined through the superpotential using Eq. (49):

$$H(\chi) = \sqrt{\frac{V_0}{3}}(1 - \exp(\mu\chi)). \quad (53)$$

To find the dependence $H(t)$, we substitute to Eq. (53) the field value (52):

$$H(t) = \sqrt{\frac{V_0}{3}} + \frac{1}{2\mu^2 t}. \quad (54)$$

Then the scale factor can be found from the definition $H(t) = \dot{a}/a$:

$$a = \exp\left(\sqrt{\frac{V_0}{3}}t\right) t^{1/2\mu^2}. \quad (55)$$

Thus we have obtained a power law-exponential evolution of the Universe which well agrees with the observational data [33].

We will use the obtained values of the potential, the field, the Hubble parameter and the scale factor for solving the set of equations (15)–(19).

The physical potential, corresponding to the chosen superpotential, as found in a standard way and has the form

$$V_{\text{Phys}}(\chi) = V_0\left[1 - 2e^{\mu\chi} + e^{2\mu\chi}\Big(1 - \frac{2\mu^2}{3}\Big)\right]. \quad (56)$$

A physical potential of such a form can be met in many solutions of Friedmannian cosmology, see, e.g., [1].

4.3. Examples of Solutions

We will seek solutions to the set of equations (15)–(19) in the same way under **Ansatz 1** and **Ansatz 2**. Here we will specify the conformal factor in the form (39).

The value of the field $\phi(t)$ (36) with the obtained value of the field $\chi(t)$ (52), the potential $W_1(\phi)$ and the Hubble parameter (54) will be the same for both Ansätze;

$$\phi(t) = \sqrt{\frac{1}{2\mu^2}}\ln t, \quad (57)$$

$$W_1(t) = V_0 + \frac{1}{2\mu^2 t^2}\left(2\sqrt{3V_0}t + \frac{3}{2\mu^2} - 1\right), \quad (58)$$

$$W_1(\phi) = V_0 + \frac{1}{2\mu^2}\exp(-2\phi\sqrt{2\mu^2}) \times \left[2\sqrt{3V_0}\exp(\phi\sqrt{2\mu^2}) + \frac{3}{2\mu^2} - 1\right]. \quad (59)$$

Using the formulas for finding the potentials with **Ansatz 1**, (44), (45), and **Ansatz 2**, (47), (46), we obtain the expressions presented in Table 2.

The obtained solutions show that solutions with the Higgs potential within the TMS TG are possible. Moreover, if we consider a flat Universe with $\epsilon = 0$, then the solutions for the potentials of the chiral fields $W_2(\phi)$ and $W_3(\psi)$ will be the same for both decompositions.

5. CONCLUSION

We have considered the possibility of a transition from gravity with a nonminimal interaction with the Higgs potential to the tensor-multi-scalar theory of

Table 2

Partitio	Solution
Ansatz 1	$h_{22}(\psi) = \epsilon\exp(-\sqrt{\frac{2V_0}{3}}\psi)\left(\frac{\psi}{\sqrt{2}}\right)^{-1/\mu^2}$ $W_2(\phi) = \frac{4AV_0}{\sqrt{2\mu^2}}\left((\phi\sqrt{2\mu^2}) + \frac{\sqrt{3}}{2\mu^2\sqrt{V_0}}\exp(-\phi\sqrt{2\mu^2}) - \frac{3}{8\mu^4V_0}\exp(-2\phi\sqrt{2\mu^2})\right)$ $W_3(\psi) = 4\sqrt{2}V_0B\left[\frac{\psi}{\sqrt{2}} - \frac{\sqrt{3}}{2\mu^2\sqrt{V_0}}\ln\frac{\psi}{\sqrt{2}} - \frac{3}{4\mu^4V_0}\frac{\sqrt{2}}{\psi}\right]$ $+ 8\epsilon\exp\left(-\sqrt{\frac{V_0}{3}}\frac{\psi}{\sqrt{2}}\right) * \frac{\psi}{\sqrt{2}}^{-1/2\mu^2}$
Ansatz 2	$h_{22}(\phi) = \epsilon\exp\left(-\sqrt{\frac{4V_0}{3}}\exp(\phi\sqrt{2\mu^2}) - \phi\sqrt{\frac{2}{\mu^2}}\right)$ $W_2(\phi) = \frac{4AV_0}{\sqrt{2\mu^2}}\left((\phi\sqrt{2\mu^2}) + \frac{\sqrt{3}}{2\mu^2\sqrt{V_0}}\exp(-\phi\sqrt{2\mu^2}) - \frac{3}{8\mu^4V_0}\exp(-2\phi\sqrt{2\mu^2})\right)$ $- \epsilon\exp\left(-2\sqrt{\frac{V_0}{3}}\exp(\phi\sqrt{-2\mu^2})\right) * (\phi\sqrt{-2\mu^2})^{-1/\mu^2}$ $W_3(\psi) = 4\sqrt{2}V_0B\left[\frac{\psi}{\sqrt{2}} - \frac{\sqrt{3}}{2\mu^2\sqrt{V_0}}\ln\frac{\psi}{\sqrt{2}} - \frac{3}{4\mu^4V_0}\frac{\sqrt{2}}{\psi}\right] + \epsilon\exp\left(-\sqrt{\frac{V_0}{3}}\frac{\psi}{\sqrt{2}}\right) \times \frac{\psi}{\sqrt{2}}^{-1/2\mu^2}$

Table 3

Partition	Solution
Ansatz 1	$M^2{}_1(\phi) = 2(3m-1)\exp\left(-\phi\sqrt{\frac{2}{m}}\right)$ $M^2{}_2(\phi) = -\frac{36A\sqrt{m^3}}{\sqrt{2}\lambda {M_p}^2}\exp\left(-\frac{4\phi}{\sqrt{2m}}\right)$ $M^2{}_3(\psi) = -\frac{48m^2B}{\lambda {M_p}^2}\psi^{-5} - \frac{4\epsilon 2^m}{c^2}m(2m+1)\psi^{-2m-2}$
Ansatz 2	$M^2{}_1(\phi) = 2(3m-1)\exp\left(-\phi\sqrt{\frac{2}{m}}\right)$ $M^2{}_2(\phi) = -\frac{36A\sqrt{m^3}}{\sqrt{2}\lambda {M_p}^2}\exp\left(-\frac{4\phi}{\sqrt{2m}}\right) - \frac{9mB\sqrt{2}}{2\lambda}\exp\left(-\frac{3\phi}{\sqrt{2m}}\right)$ $M^2{}_3(\psi) = -\frac{48m^2B}{\lambda {M_p}^2}\psi^{-5} + \frac{2\epsilon 2^m}{c^2}m(2m+1)\psi^{-2m-2}$

gravity instead of the standard transition to GR by a conformal transformation. Taking into account the additional fields, using previously developed methods for TMS TG and the CCM, allows for designing solutions in the slow-rolling approximation for the Higgs field. In what follows we will show that the obtained solutions allow us to estimate the changes of cosmological parameters induced by the additional fields.

While solving the problem, we have obtained a set of potentials related to the gravitational field. We will consider these potentials from the positions of the

Table 4

Partition	Solution
Ansatz 1	$M_1^2(\phi) = -4\sqrt{3V_0}\exp(-\phi\sqrt{2\mu^2}) + 6\mu^{-2}\exp(-2\phi\sqrt{2\mu^2}) - 4\exp(-2\phi\sqrt{2\mu^2})$
	$M_2^2(\phi) = \frac{4AV_0}{{M_p}^2\sqrt{2\mu^2}}\left(\sqrt{\frac{3}{V_0}}\exp(-\phi\sqrt{2\mu^2}) - \frac{3}{\mu^2 V_0}\exp(-2\phi\sqrt{2\mu^2})\right)$
	$M_3^2(\psi) = +\frac{4\sqrt{2}V_0 B}{{M_p}^2}\left[\frac{\sqrt{3}2\mu^2\sqrt{V_0}}{\psi}^{-2} - \frac{3\sqrt{2}}{2\mu^4 V_0}\psi^{-3}\right]$
	$+ \frac{2\epsilon V_0}{3\mu^2}\left(\frac{1}{2\mu^2} - 1\right)\exp\left(-\sqrt{\frac{V_0}{6}}\psi\right) * \left(\frac{\sqrt{2}}{\psi}\right)^{1/(2\mu^2)-2}$
Ansatz 2	$M_1^2(\phi) = -4\sqrt{3V_0}\exp(-\phi\sqrt{2\mu^2}) + 6\mu^{-2}\exp(-2\phi\sqrt{2\mu^2}) - 4\exp(-2\phi\sqrt{2\mu^2})$
	$M_2^2(\phi) = \frac{4AV_0}{{M_p}^2\sqrt{2\mu^2}}\left(\sqrt{\frac{3}{V_0}}\exp(-\phi\sqrt{2\mu^2}) - \frac{3}{\mu^2 V_0}\exp(-2\phi\sqrt{2\mu^2})\right)$
	$M_3^2(\psi) = +\frac{4\sqrt{2}V_0 B}{{M_p}^2}\left[\frac{\sqrt{3}2\mu^2\sqrt{V_0}}{\psi}^{-2} - \frac{3\sqrt{2}}{2\mu^4 V_0}\psi^{-3}\right]$
	$+ \frac{\epsilon V_0}{12\mu^2}\left(\frac{1}{2\mu^2} - 1\right)\exp\left(-\sqrt{\frac{V_0}{6}}\psi\right) * \frac{\sqrt{2}}{\psi}^{1/(2\mu^2)-2}$

standard approach of particle theory and determine the masses corresponding to the scalar particles of TMS TG in a standard way.

We will find the scalar field mass from the second-order derivative of the potential by the formula

$$M^2 = \left(\frac{d^2W}{d\phi^2}\right)_{\phi=0} \tag{60}$$

Table 3 presents the results of computing the second-order derivatives for the obtained solutions for Case 4.1.

Let us note that the mass M_1 is completely determined by the degree of expansion (the parameter m). The mass of M_2 is determined by both the degree of expansion of the Universe, the Higgs potential parameter, λ, and the Planck mass M_p. The mass of the field ψ M_3 tends to infinity for any parameters, except for evolution with $a(t) \propto t^{3/2}$ under the relation for constants $-9Bc^2 = \lambda M_p^2 \epsilon 2^{5/2}$. With the specified ratio, $M_3 = 0$. The situation is the same for Ansatz 2.

Table 4 presents the results of calculation of the second-order derivatives for the obtained solutions in Case 4.2.

In this case, the masses M_1 and M_2 are determined by both the degree of expansion of the Universe and the parameters of the Higgs potential λ, ξ as well as the Planck mass (it is involved in V_0 and μ).

FUNDING

The study has been carried out under financial support of RFBR under the research project No. 20-02-00280-A. S.V. Chervon is grateful for the support of the Program of Competitive Growth of Kazan Federal University.

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